ENGR 7 W03 HW3 solutions

(Total 100 points)

1. Problem 9, page 228, chapter 9 (20 pts)

(a) (6 pts)

Since the population standard deviation σ is unknown, we use the t distribution with 13 df rather than the normal distribution. A two-sided 95% confidence interval for μ is:

$$(29.6 - 2.160\frac{3.6}{\sqrt{14}}, 29.6 + 2.160\frac{3.6}{\sqrt{14}}) = (27.5, 31.7)$$

(b) (2 pts)

The length of this interval is 31.7 - 27.5 = 4.2 weeks.

(c) (6 pts)

Since the interval is centered around the sample mean $\bar{x} = 29.6$ weeks, we are interested in the sample size necessary to produce the interval:

$$(29.6 - 1.5, 29.6 + 1.5)$$

Since the population standard deviation σ is known, we use the normal distribution. We know that the 95% confidence interval is of the form:

$$(29.6 - 1.96\frac{3.6}{\sqrt{n}}, 29.6 + 1.96\frac{3.6}{\sqrt{n}})$$

To find n, we solve the equation:

$$1.5 = \frac{1.96(3.6)}{\sqrt{n}} \Longrightarrow n = 22.1$$

So a sample of size 23 is required.

(d) (6 pts)

Here we are interested in the sample size necessary to produce the interval:

$$(29.6 - 1, 29.6 + 1)$$

The 95% confidence interval is of the form:

$$(29.6 - 1.96\frac{3.6}{\sqrt{n}}, 29.6 + 1.96\frac{3.6}{\sqrt{n}})$$

To find n, we solve the equation:

$$1 = \frac{1.96(3.6)}{\sqrt{n}} \Longrightarrow n = 49.8$$

So a sample of size 50 is required.

2. Problem 11, page 229, chapter 9 (20 pts)

(a) (8 pts)

Because the population standard deviation is unknown, we use the t distribution with 7 df rather than the normal distribution.

The sample mean calcium level is $\bar{x}_c = 3.14$ mmol/l and the sample standard deviation is $s_c = 0.51$ mmol/l. [4 pts]

A one-sided lower 95% confidence bound for the true mean calcium level μ_c in mmol/l is: [4 pts]

$$3.14 - 1.895\frac{0.51}{\sqrt{8}} = 2.80$$

(b) (8 pts)

The sample mean albumin level is $\bar{x}_a = 40.4$ g/l and the sample standard deviation is $s_a = 3.0$ g/l. [4 pts]

A one-sided lower 95% confidence bound for the true mean albumin level μ_a in g/l is: [4 pts]

$$40.4 - 1.895\frac{3.0}{\sqrt{8}} = 38.4$$

(c) (4 pts)

The lower 95% confidence bound for the mean calcium level does not lie within the normal range of values; this suggests that calcium levels are elevated for this group. There is no evidence that albumin levels differ from the normal range.

3. (18 pts)

(a) (6 pts)

Since the population standard deviation σ is unknown, we use the t distribution with 399 df (which turns out to be essentially the same as the normal distribution). A 98%

confidence interval for μ is:

$$(10.4 - 2.327\frac{4.2}{\sqrt{400}}, 10.4 + 2.327\frac{4.2}{\sqrt{399}}) = (9.9, 10.9)$$

(b) (3 pts)

Since 10 is (barely) included in the 98% confidence interval, so with 98% confidence, I would not recommend that the consumer group issue a report that the mean tire pressure is seriously underinflated.

(c) (9 pts)

A 90% confidence interval for μ is: [6 pts]

$$(10.4 - 1.645\frac{4.2}{\sqrt{400}}, 10.4 + 1.645\frac{4.2}{\sqrt{399}}) = (10.1, 10.7)$$

Since 10 lies to the left of the 90% confidence interval, so with 90% confidence, I would recommend that the consumer group issue a report that the mean tire pressure is seriously underinflated. [3 pts]

4. Problem 9, page 254, chapter 10 (18 pts)

(a) (2 pts)

The null hypothesis of the test is: $H_0 : \mu = 74.4$

(b) (2 pts)

The alternative hypothesis of the test is: $H_A : \mu \neq 74.4$

(c) (10 pts)

The test statistic is: [6 pts]

$$z = \frac{\bar{x}_d - \mu_0}{\sigma_d/\sqrt{n}} = \frac{84 - 74.4}{9.1/\sqrt{n}} = 3.34$$

The area to the right of z = 3.34 is less than 0.001, so is the area to the left of z = -3.34, so the p-value is less than 0.002. [2 pts]

Since $p < 0.05$, so we reject H_0. [2 pts]

(d) (2 pts)

We conclude that the mean diastolic blood pressure for the population of femail diabetics between the ages of 30 and 34 is not equal to 74.4 mm Hg. In fact, it is higher.

(e) (2 pts)

Since p $<$ 0.01, the conclusion would have been the same.

5. Problem 10, page 255, chapter 10 (16 pts)

(a) (4 pts)

The null hypothesis of the test is: $H_0 : \mu \geq 7250$ (or $\mu = 7250$)

The alternative hypothesis of the test is: $H_A : \mu < 7250$

(b) (10 pts)

Since the population standard deviation is unknown, we use the t-test rather than the z-test. The test statistic is: [6 pts]

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{4767 - 7250}{3204/\sqrt{15}} = -3.0$$

For a t distribution with 14 df, 0.0005 $<$ p $<$ 0.005. [2 pts]

Since p $< \alpha$, we reject H_0. [2 pts]

(c) (2 pts)

We conclude that the mean white blood cell count of humans infected with *E. canis* is lower than $7250/mm^3$, the mean of the general population.

6. (8 pts)

Since the confidence interval is centered around the sample mean $\bar{x}$, we are interested in the sample size necessary to produce the interval:

$$(\bar{x} - 3, \bar{x} + 3)$$

Since the population standard deviation σ is known (to be 13), we use the normal distribution. We know that the 99% confidence interval is of the form:

$$(\bar{x} - 2.58\frac{13}{\sqrt{n}}, \bar{x} + 2.58\frac{13}{\sqrt{n}})$$

To find n, we solve the equation:

$$3 = \frac{2.58(13)}{\sqrt{n}} \implies n = 124.99$$

So a sample of size 125 is required.