April 28, 2005 Lecture Notes Measurement Error, Probability, Models for Means

Measurement error: No matter how carefully made, any measurement, if repeated, could come out differently. Why?

Basic measurement error equation:

$$\begin{pmatrix}\text{each individual}\\ \text{measurement}\end{pmatrix} = \begin{pmatrix}\text{exact "true"}\\ \text{value}\end{pmatrix} + \begin{pmatrix}\text{bias}\end{pmatrix} + \begin{pmatrix}\text{a random error}\end{pmatrix}$$

bias: systematic error – 0 in this case study

Random errors fluctuate around 0 – mean of random errors is 0 + SD = 0.2

Case Study 9:

$$\begin{pmatrix}\text{each individual}\\ \text{measurement}\end{pmatrix} = \begin{pmatrix}\text{exact "true"}\\ \text{value}\end{pmatrix} + \begin{pmatrix}\text{bias}\end{pmatrix} + \begin{pmatrix}\text{a random}\\ \text{error}\end{pmatrix}$$

observable – vary around truth with a SD of 2

3.7 = potassium (K) reading #1 = 3.8 + 0 + (−0.1)
4.0 = potassium (K) reading #2 = 3.8 + 0 + (+0.2)

unobservable
mean of random errors is 0

K = symbol for potassium

population	sample	imaginary data set
conceptual: all possible K readings for you	the observed K readings	possible means $\bar{y}$

K concentration: $N = \infty$, $\begin{bmatrix} \vdots \end{bmatrix}$ $\xrightarrow{\text{IID}}$ K: $\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$ n $\longrightarrow$ $\begin{bmatrix} 3.6 \\ 4.0 \end{bmatrix}$ avg, avg, ∞

mean $\mu = 3.8$
SD $\sigma = 0.2$

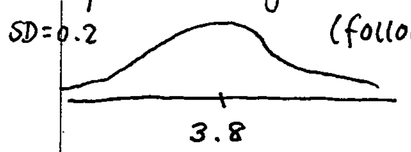
mean $\bar{y} = ?$ (ex: 3.6)

need to know:
long run mean: 3.8

long run SD: 0.1

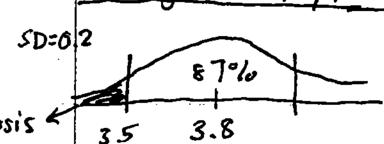
long run histogram:
(on next page)

Population histogram



(follows normal curve)

histogram of y_1 (one draw)



$\frac{3.5 - 3.8}{0.2} = -1.5$ (87% according to chart)

$\frac{100\% - 87\%}{2} = 6.5\%$ chance of misdiagnosis

$n=4$ (we want 4 readings)

P(misdiagnosis w/ $n=4$) = P($\bar{y} < 3.5$ with $\bar{y}$ based on $n=4$ readings)

Long Run Mean = Expected value of $\bar{y}$ = $E(\bar{y}) = EV = E_{IID}(\bar{y}) = \mu = 3.8$

Long Run SD = Standard Error of $\bar{y}$ = $SE(\bar{y}) = SE = SE_{IID}(\bar{y})$

(noise of uncertainty of $\bar{y}$ as estimate of μ)

N - nothing to do w/SE

μ - nothing to do w/SE

σ - $\sigma\uparrow$ $SE(\bar{y})\uparrow$

n - $n\uparrow$ $SE(\bar{y})\downarrow$

most important formula in stats: $\boxed{SE_{IID}(\bar{x}) = \frac{\sigma}{\sqrt{n}}}$

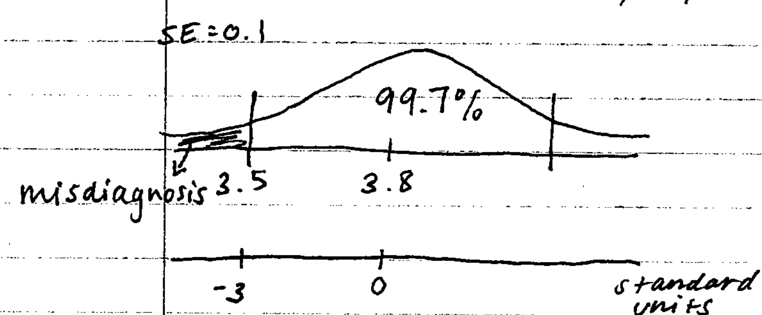
(uncertainty about μ goes down as $n\uparrow$ but only at a rate of $\sqrt{n}$: to cut SE in half, I need to quadruple (x4) sample size n)

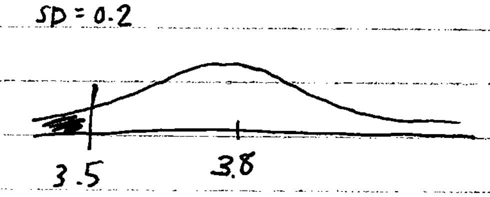
thus, $SE_{IID}(S) = \sigma\sqrt{n}$ (sums) $\quad SE_{IID}(\bar{y}) = \frac{\sigma}{\sqrt{n}}$ (means (averages))

$$SE(\bar{y}) = \frac{\sigma}{\sqrt{n}} = \frac{0.2}{\sqrt{4}} = 0.1$$

The long Run SD is 0.1

Long Run Histogram of $\bar{y}$ w/ $n=4$: w/ $n=1$:





6.5% chance of misdiagnosis (from previous page)

$\frac{3.5 - 3.8}{0.1} = -3$ $\qquad$ $\frac{100\% - 97\%}{2} = 0.15\%$ chance of misdiagnosis

according to chart on p. 33, 99.7% in middle

cost	n (# of tests)	P(incorrect diagnosis)
\$25	1	6.5%
\$100	4	1.5%

here, the consequences of misdiagnosis are relatively mild (eating banannas when you don't need to)
↳ decision theory : utility : worth you attach to correct diagnosis (you decide if it is worth the extra \$75 to have a better chance of correct diagnosis)
(End of Case Study 9)

Why does it help to take an average of $(n>1)$ readings instead of just 1?

no bias:

observation 1 = truth + random error 1
observation 2 = truth + random error 2
⋮ ⋮ ⋮
observation n = truth + (average of n random errors)
fluctuate around 0 w/ SD σ

ex: $\frac{(-0.1)+(0.2)+\ldots+(-0.3)+(+0.1)}{n}$ = cancelation of ⊕ + ⊖ errors
$SE(\bar{y}) = \frac{\sigma}{\sqrt{n}}$ measures likely size of mean of n errors

w/bias:

observation 1 = truth + bias + random error 1
⋮ ⋮ ⋮ ⋮
observation n = truth + bias + (average of n random errors)
gets to 0 as $n \to \infty$
however, bias will increase, making data worse

so, w/bias, (average of n observations) = truth + bias (which only gets worse)