1

5/10/05

Pitfalls of inference

① How good is SE (in general) as a give-or-take for $\bar{y}$ or $\hat{p}$?

$$\text{Observation}_1 = \text{truth} + \text{bias} + \text{random error}_1$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\underline{\text{Obs.}_n = \text{truth} + \text{bias} + \text{random error}_n}$$ (equivalent to taking the mean)

$$\bar{y} \text{ or } \hat{p} = \text{truth} + \text{bias} + (\text{avg. of } n \text{ random errors})$$

rewritten

$\hat{SE}$ measures the likely size of

$$\bar{y} \text{ or } \hat{p} - \text{truth} = \text{bias} + (\text{avg. of } n \text{ random errors})$$

∴ if there is no bias, $\hat{SE}$ = Likely amount by which $\bar{y}$ or $\hat{p}$ will differ from the truth <u>but</u> if there is BIAS, (math fact!) the

$$\left(\begin{array}{l}\text{Likely amount by}\\ \text{which } \bar{y} \text{ or } \hat{p} \text{ will}\\ \text{differ from truth}\end{array}\right) = \sqrt{(\text{bias})^2 + (\hat{SE})^2}$$

Special case: no bias → this equation reduces to $\hat{SE}$ ✓

if Bias is present, $\hat{SE}$ underestimates $\sqrt{(\text{bias})^2 + (\hat{SE})^2}$

⟹ if (substantial) bias present in sampling method, $\sqrt{(\text{bias})^2 + (\hat{SE})^2}$ can be a lot bigger than $\hat{SE}$ (understates your real margin of error (ex. 2 in H/W 4)

② How inference can fail completely:

ex. pop → sample

our income

[\$] → [\$] $n = 140$

pop: mean: $\mu = ?$, SD: $\sigma = ?$

sample: mean $\bar{y} = \$60{,}000$, SD $s = \$30{,}000$

$$95\% \text{ C.I. for } \mu: \ \bar{y} \pm 2\hat{SE}(\bar{y}) = \bar{y} \pm 2\frac{s}{\sqrt{n}} = \$60{,}000 \pm 2\left(\frac{\$30000}{\sqrt{140}}\right)$$

$$= \$60{,}000 \pm 2(\$2535) = \$60{,}000 \pm \$5070 \approx \$60{,}000 \pm \$5000$$

\$55,000 — \$60,000 — \$65,000

Q_1: 95% C.I. for what? A_1: For μ
Q_2: what is μ? A_2: pop. mean
Q_3: what's the pop., and do we actually have a random sample from it?

Possible populations:

P_1: everybody enrolled in AMS 5 this quarter who came to class today
if this is the pop. of interest then we have Ø uncertainty about μ, and the right answer for μ is $\bar{y}$ = \$60,000 ± Ø
(in this case sample = pop. and there is no uncertainty left in pop.)

P_2: All students enrolled at UCSC in Spring 2005
this pop. is ok, and sample ≠ pop., but you guys might be a biased sample from this pop. and size (and even direction) of this bias are hard to pin down
So \$60,000 ± \$5,000 is probably too narrow to reflect our real uncertainty about mean μ of P_2, etc...

Punchline:
if you can't identify an interesting pop. from which your data are arguably like a random sample, then don't do inference (C.I. or significance tests). Two basic ways inference can fail: (#2,4 HW4)

Conceptual failures: whole idea is silly b/c sample not related randomly to the right pop. (example above)

technical failures: (example - if data show time trend)