

①  
5/10/05

### Pitfalls of inference

① How good is SE (in general) as a give-or-take for  $\bar{y}$  or  $\hat{p}$ ?

Observation<sub>1</sub> = truth + bias + random error,  
⋮  
⋮  
⋮  
⋮

$$\frac{\text{Obs.}_n = \text{truth} + \text{bias} + \text{random error}_n}{\bar{y} \text{ or } \hat{p} = \text{truth} + \text{bias} + (\text{avg. of } n \text{ random errors})} \quad \leftarrow \begin{matrix} \text{(equivalent to} \\ \text{taking the mean)} \end{matrix}$$

rewritten

$\hat{SE}$  measures the likely size of

$$\bar{y} \text{ or } \hat{p} - \text{truth} = \text{bias} + (\text{avg. of } n \text{ random errors})$$

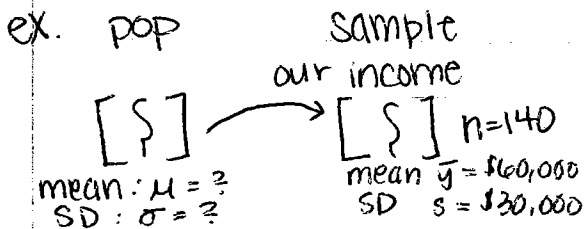
∴ if there is no bias,  $\hat{SE}$  = Likely amount by which  $\bar{y}$  or  $\hat{p}$  will differ from the truth but if there is BIAS, (math fact!) the

$$\left( \begin{matrix} \text{Likely amount by} \\ \text{which } \bar{y} \text{ or } \hat{p} \text{ will} \\ \text{differ from truth} \end{matrix} \right) = \sqrt{(\text{bias})^2 + (\hat{SE})^2}$$

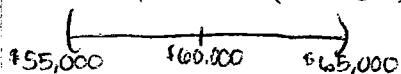
Special case: no bias → this equation reduces to  $\hat{SE}$  ✓

if Bias is present,  $\hat{SE}$  underestimates  $\sqrt{(\text{bias})^2 + (\hat{SE})^2}$   
⇒ if (substantial) bias present in sampling method,  $\sqrt{(\text{bias})^2 + (\hat{SE})^2}$  can be a lot bigger than  $\hat{SE}$  (understates your real margin of error (ex. 2 in HW 4))

② How inference can fail completely:



$$95\% \text{ C.I. for } \mu: \bar{y} \pm 2 \hat{SE}(\bar{y}) = \bar{y} \pm 2 \frac{s}{\sqrt{n}} = \$60,000 \pm 2 \left( \frac{\$30,000}{\sqrt{140}} \right) \\ = \$60,000 \pm 2(\$2535) = \$60,000 \pm \$5070 \approx \$60,000 \pm \$5000$$



Q<sub>1</sub>: 95% C.I. for what? A<sub>1</sub>: For  $\mu$

Q<sub>2</sub>: What is  $\mu$ ? A<sub>2</sub>: pop. mean

Q<sub>3</sub>: What's the pop., and do we actually have a random sample from it?

Possible Populations:

P<sub>1</sub>: everybody enrolled in AMS 5 this quarter who came to class today

if this is the pop. of interest then we have  $\emptyset$  uncertainty about  $\mu$ , and the right answer for  $\mu$  is  $\bar{y} = \$60,000 \pm \emptyset$  (in this case sample = pop. and there is no uncertainty left in pop.)

P<sub>2</sub>: All students enrolled at UCSC in Spring 2005

this pop. is OK, and sample  $\neq$  pop., but you guys might be a biased sample from this pop. and size (and even direction) of this bias are hard to pin down

So  $\$60,000 \pm \$5,000$  is probably too narrow to reflect our real uncertainty about mean  $\mu$  of P<sub>2</sub>, etc...

punchline:

if you can't identify an interesting pop. from which your data are arguably like a random sample, then don't do inference (C.I. or significance tests). Two basic ways inference can fail: (#2,4 #1/4)

Conceptual failures: whole idea is silly b/c sample not related randomly to the right pop. (example above)

technical failures: (example - if data show time trend)

