26 Apr ①

AMS 5

this time: probability models for sums

next time: prob. models for means

read: DD ch. 11, FPP ch 6, 18

hints on midterm: (1) problem 2 is like gender/MLP from class & homework 3; (2) problem (5) is like roulette from class, keno (ch. 9 from DD), & extra discussion section problem (pyx) this week

sample SD

$$s = \sqrt{\frac{(y_1-\bar{y})^2+(y_2-\bar{y})^2+\dots+(y_n-\bar{y})^2}{n-1}}$$

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

mean $\bar{y}$

SD s

I expect on each spin of roulette wheel to lose about a nickel, give or take about \$5.76

sum S of $n = 1{,}000$ IID draws from this pop. is like my net gain at end of 1,000 gambles with strategy (A)

$P\left(\text{coming out ahead with strategy A}\right) = P(\$ > \$0) = ?$

$(+\$35)$

$P\left(\text{coming out ahead on any single bet on a single \#}\right) = \dfrac{1}{38} \doteq 2.6\%$

"most likely value for $\$$" in 1000 gambles I expect to win $\dfrac{1000}{38} \doteq 26$ times

$$\underbrace{\underbrace{974 \text{ losses of } \$1}_{-\$974}, \; \underbrace{26 \text{ wins of } \$35}_{+\$910}}_{-\$64} :$$

$= EV$

long run mean of $\$$ = expected value of $\$$ $= E(\$) = E_{IID}(\$)$

$= \left(\#\text{ draws}\right)\left(\text{pop mean}\right) = n\mu$

$= (1000)(-\$0.053) = -\$53$

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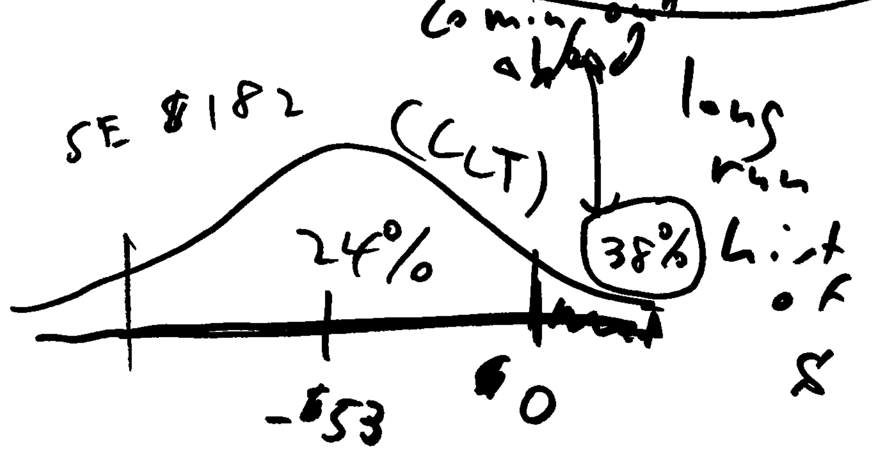
at end of 1000 gambles with A I expect to be behind by about $53.

long run SD of S = standard error of S = SE



$$\frac{\$0-(-\$53)}{\$182} = +0.3$$

$$= SE(S) = SE_{IID}(S) = \underset{\text{sum}}{\overset{SE}{\text{of}}} = \frac{\sigma\sqrt{n}}{1} = \sigma\sqrt{n}$$

N	✗		
μ	✗		
σ	↑	SE	↑
n	↑	SE	↑

$= (\$5.76)\sqrt{1000}$

$\doteq \$182$

IID = independent identically distributed

I expect to be behind by about $53 (EV) at end of 1000 spins with (A), give or take about $182 (SE)

CLT (Central Limit Theorem)

the long run hist of sum (or mean)

of (n) IID draws from a pop will look a lot like the normal curve provided (n) is big enough; the closer the pop. hist. is to normal curve to begin with, the smaller (n) can be to get a good normal approximation (for sum or mean)

P(coming out ahead (A)) = 38%

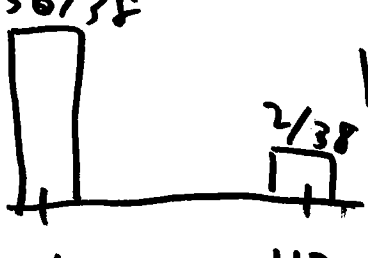
(B) (1, 2)

pop
possible gains
my net gain \$

1	+17
2	+17
3	−1
4	−1
⋮	⋮
36	−1
0	−1
00	−1

mean $\mu = -\$0.053$

SD $\sigma = 84.02$



IID →

sample
the observed gain
my net gain \$

−1
−1
⋮
+17
⋮

n = 1,000

sum S = ? (ex. −\$48)

IID

n = 1000

sum S = ? (ex. +\$10)

I.D.
possible values of S
\$

−48
+10
⋮

a S ↑ ∞ ↓

long run mean — $E(S) = -\$53$

long run SD — $SE(S) = \$127$

long run hist.