21 Apr (1)

AMS 5

this time: probability, conditional probability

next time: expected value, standard error, central limit theorem, starting with

read: (D) ch. 9,10) FPP ch 18

homework 3 (due Thu 26 Apr), please put your discussion section code on your hwk paper

from now on, hwk will be handed back in section

P(1 or more T-S kids in family of 5) =

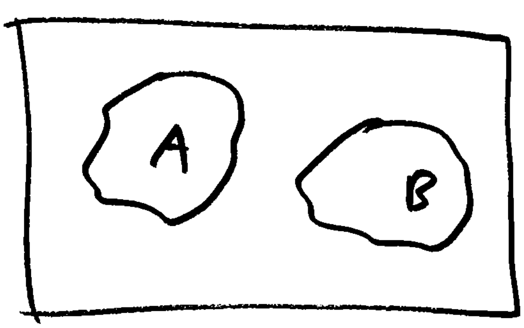
P(exactly 1 or exactly 2 or ... or exactly 5)

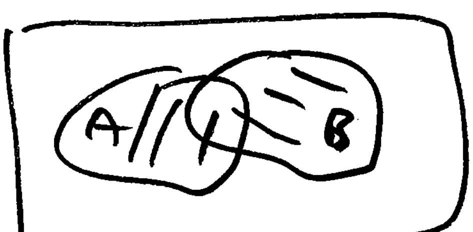
= 100% − P(no T-S babies)

P(no T-S kids) = P(1st kid not T-S and 2nd kid not T-S and ... and 5th kid not T-S)

need rules for and, or, not, given

or: P(A or B) = P(A) + P(B)





P(A or B)

= P(A) + P(B) − P(A and B)

addition rule for OR

and (conditional probability) (IID = independent identically distributed) ②

pop $\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix}$ $\xrightarrow{\text{at random}}$ with replacement (IID) sample $\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$ $n=2$

(multiply)

$P(\underline{Y_1 = 2 \text{ and } Y_2 = 2}) \overset{?}{=} P(Y_1 = 2) \overset{?}{\cdot} P(Y_2 = 2)$

with

$$\begin{array}{cc|ccc} & & & Y_2 & \\ & & 1 & 2 & 9 \\ \hline & 1 & (1,1) & (1,2) & (1,9) \\ Y_1 & 2 & (2,1) & (2,2) & (2,9) \\ & 9 & (9,1) & (9,2) & (9,9) \end{array}$$

ELM applies

$P(Y_1 = 2 \text{ and } Y_2 = 2) = \frac{1}{9}$

$P(Y_1 = 2) = \frac{1}{3} = \frac{3}{9}$

$P(Y_2 = 2) = \frac{1}{3} = \frac{3}{9}$

$P(Y_1 = 2 \text{ and } Y_2 = 2) = P(Y_1 = 2) \cdot P(Y_2 = 2)$

$P(A \text{ and } B) = P(A) \cdot P(B) \leftarrow$ special product rule for and

pop $\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix}$ $\xrightarrow{\text{at random}}$ without replacement sample $\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$ $n=2$

$P(Y_1 = 2 \text{ and } Y_2 = 2) = 0$

without

			Y_2	
		1	2	9
	1	X	(1,2)	(1,9)
Y_1	2	(2,1)	X	(2,9)
	9	(9,1)	(9,2)	X

$$P(Y_1 = 2 \text{ and } Y_2 = 2) = 0 = \frac{0}{6}$$

ELM

$$\overset{?}{=} \underbrace{P(Y_1 = 2)}_{} \cdot P(Y_2 = 2)$$

$$\left(\frac{1}{3} = \frac{2}{6}\right) \cdot \left(\frac{2}{6} = \frac{1}{3}\right)$$

$$= \frac{1}{9}$$

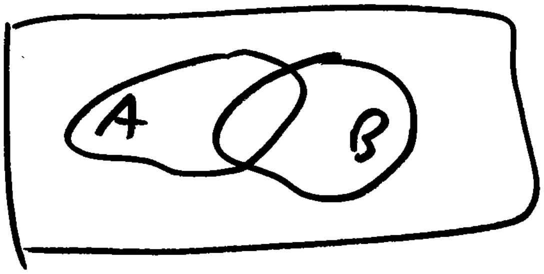
$$\neq 0$$

conditional probability

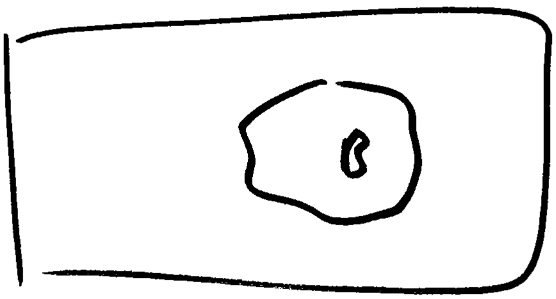
$$P(B \text{ given } A)$$

$$= P(B \mid A)$$

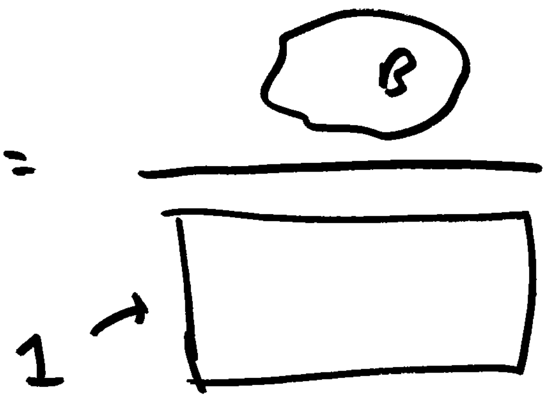
"given"

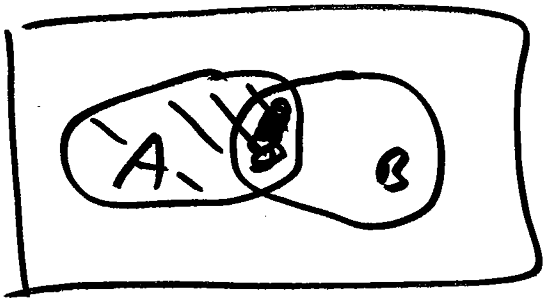


$$P(B \text{ given } A) = ?$$

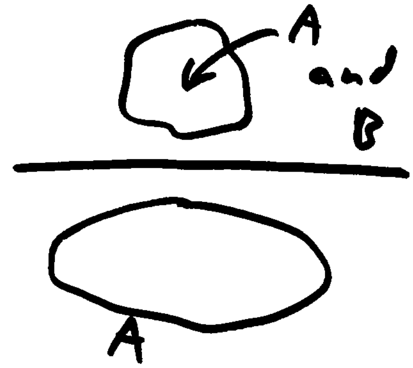


$P(B) =$





$P(B \text{ given } A) =$



def: (Rev. T. Bayes (1740))

$$P(B \text{ given } A) = \frac{P(A \text{ and } B)}{P(A)}$$

mult both sides by $P(A)$:

$$P(A \text{ and } B) = P(A) \cdot P(B \text{ given } A)$$

(general product rule for and)

$$P(A \text{ and } B) = P(B) \cdot P(A \text{ given } B)$$

$$\phantom{P(Y_1=2 \text{ and } Y_2=2) = } \quad (A) \qquad\qquad (B \mid A)$$

$$P(\underbrace{Y_1=2}_{A} \text{ and } \underbrace{Y_2=2}_{B}) = P(Y_1=2) \cdot P(Y_2=2 \mid Y_1=2)$$

$$\big(= P(Y_2=2) \cdot P(Y_1=2 \mid Y_2=2)$$

(too hard)

$$= P(Y_1=2) \cdot P(Y_2=2 \mid Y_1=2) \leftarrow \begin{bmatrix} 1 \\ 9 \end{bmatrix}$$

$$= \frac{1}{3} \cdot 0 = 0 \checkmark$$

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suppose knowledge that A is true <u>does not</u> change the chance that B is true & vice versa – then A and B are <u>independent</u> & in that case

$$P(A \text{ and } B) = P(A) \cdot P(B \text{ given } A)$$

$$\overset{\text{if indep}}{=} P(A) \cdot P(B) \quad (\text{special rule})$$