Adapted differentiators for image motion estimation

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The author describes the design and use of adapted differentiators for improving the accuracy of differential-based optical flow algorithms.

Introduction: Differential-based algorithms for the measurement of the motion field in an image sequence have been widely studied [1]. They are based on the estimation of the spatial-temporal partial derivatives of the image. A first-stage lowpass filter is always present in such systems, to remove noise and to reduce the effect of aliasing and quantisation.

In the literature, simple 3-tap or 5-tap differentiators to compute the spatial-temporal derivatives have been considered. We propose the use of larger size differentiators, which are adapted to the first-stage lowpass filter according to a weighted least-squares criterion. Basically, the adapted differentiators are forced to behave like 'ideal' differentiators in the passband of the lowpass filter, i.e. where most of the signal energy is preserved. The accuracy of the derivative estimation, and thus of the measured optical flow, is improved with respect to nonadapted differentiators. We also derive a closed-form solution to the approximation problem when the lowpass filter is FIR.

Theory: The low-level section of any differential-based optical flow algorithm is composed of a (spatial-temporal) lowpass filter, followed by the partial derivative estimators. It is important to realise that these two parts must be separated, for the following reason. Let $I(x,y,t)$ be the brightness signal as a function of space and time. It can be easily shown [2] that in regions where the motion field $I(x,y,t)$ is uniformly translational, the motion field of any filtered version of $I(x,y,t)$ corresponds to that of $I(x,y,t)$. In other words, the action of the lowpass filter is irrelevant in terms of optical flow measurement. If, on the other hand, a lowpass action were exhibited by the spatial-temporal differentiators independently, one would measure a motion field which is different from the original one (note that it is common practice to design differentiators with a lowpass behaviour, to avoid the amplification of noise in the highpass region). Hence, the lowpass action should be performed by the first-stage filter entirely, while the three differentiators should be 'as close as possible' to ideal differentiators.

However, the ideal differentiator cannot be realised by FIR (nonrecursive) digital filters. We propose to approximate the ideal differentiator using FIR filters that are adapted to the first stage lowpass filter, in the following sense. Let $H(\omega)$ be the frequency response of the lowpass filter in the direction along which the partial derivative is to be computed (we assume that the lowpass filter is separable in space-time). The frequency response of the ideal digital differentiator (periodic with period 2π) is

$$D_{id}(\omega) = j\omega \qquad -\pi \leq \omega < \pi \tag{1}$$

and we would like to approximate it with an FIR filter $D(\omega)$. Clearly, it is convenient that the approximation error $E(\omega) = D(\omega) - D_{id}(\omega)$ be smaller in regions where the signal is expected to conserve more energy after the lowpass filter. We propose to use a weighted least squares criterion, i.e. to minimise the quadratic norm of $H(\omega)E(\omega)$, defined as

$$||H(\omega)E(\omega)||^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |H(\omega)(D(\omega) - D_{id}(\omega))|^2 \, d\omega \tag{2}$$

If $H(\omega)$ is FIR, a closed-form solution to the minimisation task exists (more details may be found in [2]). Let N_H and N_D be the lengths of the filters $H(\omega)$ and $D(\omega)$, respectively. The integrand in eqn. 2 can be rewritten as

$$|H(\omega)D(\omega) - D_{id}(\omega)H(\omega)|^2 \tag{3}$$

Let $\mathbf{H}$ be the space of length-(N_H+N_D–1) FIR filters, and let $\mathbf{H}_1$ be the N_H-dimensional subspace of $\mathbf{H}$ composed of the filters of the form $H(\omega)B(\omega)$, where $B(\omega)$ is any length-N_D FIR filter. We are looking for the element of $\mathbf{H}_1$ which minimises the quadratic distance to $D_{id}(\omega)H(\omega)$. A two-step solution may be devised:
1. Find the filter $A(\omega)$ of $\mathbf{H}$ which minimises the quadratic distance to $D_{id}(\omega)H(\omega)$.
2. Find the length-N_D filter $D(\omega)$ such that $H(\omega)D(\omega) \in \mathbf{H}_1$ minimises the quadratic distance to $A(\omega)$.

The first minimisation is obtained simply by windowing the impulse response of $D_{id}(\omega)H(\omega)$ [3], while the second one is a least-squares problem involving a Toeplitz matrix (which represents the filtering with $H(\omega)$).

In closing, let us note that the use of weighted least squares differentiators (although in a different context and without the derivation of a closed-form solution) has also been considered in [4].

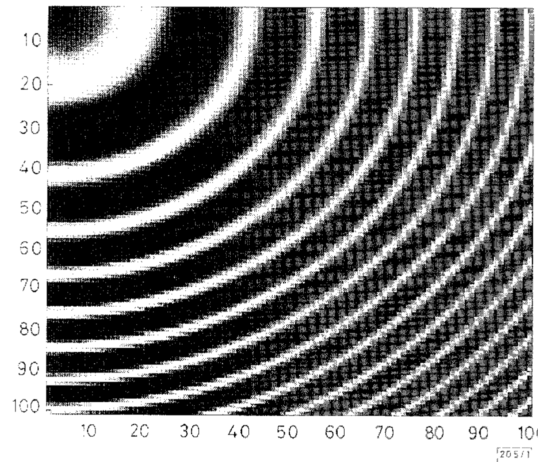

Fig. 1 *Zone plate test pattern used for test*

Experiments: For our test, we have implemented the Lucas-Kanade algorithm [1] using adapted differentiators. The lowpass filter was a separable Gaussian, with standard deviation $\sigma_t = 1$ along the temporal direction, and $\sigma_{xy} = 4/3$ along the horizontal-vertical direction. The impulse responses of the adapted differentiators with $N_D = 7$ were (0.04, –0.23, 0.84, 0, –0.84, 0.23, –0.04) along the temporal direction, and (0.09, –0.39, 1.02, 0, –1.02, 0.39, –0.09) along the horizontal-vertical direction. We tested the algorithm on a translating zone plate image, a standard test pattern characterised by a very rich spectral content (see Fig. 1). The optical flow estimation error was evaluated in each pixel using the angular measure introduced in [1]. For the pattern translating by 2.5 pixels/frame and simple forward-backward differentiators, the error, averaged over all pixels, was $m_e = 34.4°$, with standard deviation $\sigma_e = 38.58$. Using the adapted differentiators, we obtained $m_e = 1.9°$ and $\sigma_e = 6.0$. The improvement in the system's accuracy is apparent. Note, however, that the performances of optical flow systems depend on a number of other factors beyond derivative estimation, such as aperture effect, brightness changes and noise.

Conclusion: We have described the design and use of adapted differentiators for optical flow systems. The accuracy of the velocity measurements may be effectively improved using adapted differentiators in differential-based schemes.

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