Explorations in Scientific Thinking:

A Systems Theoretic Approach

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2 Characteristics and Laws of Systems

Chapter 1

Introduction to Systems and Modeling

1.1 Variables and States

Consider the following scenario of "the robot and the TV". In this scenario a robot, that only knows how to run programs, must be programmed to travel from one room, through an open door into an adjacent room, turn on the TV that is in front of a chair, and then sit down in the chair. A common wall separates the two rooms. In the common wall there is a door.

To study the system described below, we will keep track of certain system characteristics that interest us. These characteristics must be observable, measurable and must also change. Another word for these characteristics is *variable.* Usually, a system will have a group of variables that we are keeping track of. At any given point in time, these variables will all be in a certain condition, or have a certain value. This collection of system variable values (or conditions) is also referred to a *system state*, similar to the expression: "a state of affairs". Using the vocabulary of variables, and states we can thus begin to describe the "robot and TV" scenario as a system (the RTS). Refer to Figure 1.1.

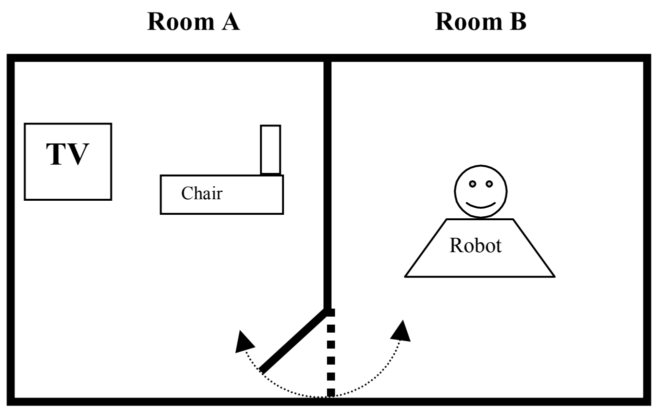


Figure 1.1, Diagram of Robot and TV System

The robot can occupy only one room (R) at a time. The room variable can have only two states, either Room **A** (which has the TV and chair in it) or Room **B**, which has no furniture in it. The door (D) variable can only have two states: either open (**O**) or closed (**C**). The chair (C) variable, can only have two states also and is either occupied (**P**) or empty (**E**). The TV variable (T), is either in the state of being on (**N**) or off (**F**). Thus, these variables and state variable values can be summarized as follows:

R: {A,B}
D: {O,C}
C: {P,E}
T: {N,F}

1.2 States: Start and End

For this example, let's say that the desired final, or *end*, state is to have the robot in room A sitting down in front of the TV with the TV "ON". The final and "desired" end state can be summarized also as:

R = A: room A
D = O: door is open so robot can go room to room,
C = P: robot is sitting
T = N: TV is "ON"

To continue with our description of this system, we would need to begin with an initial, or *start*, state. For example, the robot could be in room B, the door could be open, the chair empty and the TV off. Thus, this initial state could be summarized as:

R = B
D = O
C = E
T = F

1.3 Operators

Lets say that, from this start state, our simple system needs to transition into the end state described above. We could enumerate all of the possible states that the system could transition through as a sequence of states that comprise a series of steps. To keep track of the transitions, or steps, we could name each of the transition rules that specify how each individual variable can change. For example, we need a transition rule for specifying that the room variable can change state. Another word for describing a transition rule is *operator*. So, the operator for the room transition rule could be CR, for "change room state". To account for the other variables, the operators could be called: CD, for "change door state"; CC, for "change chair state"; and CT, for "change TV state".

To summarize what we have so far for our abstract, symbolic representation of the RTS: the variables, state variable values, and operators, we have:

State Variable	State Variable Values	Operators
R	A, B	CR
D	O, C	CD
C	P, E	CC
T	N, F	CT

1.4 System Constraints: Possibility and Reachability

Returning to the notion of the system beginning with some start state and then changing, by action of one or more operators into another state, we now encounter the notion of what is both, possible or not possible. For example, the robot must be in room A in order to turn the TV ON. Alternatively, the robot cannot turn on the TV if it is room B. Similarly, the robot must be in room A in order to sit in the chair and could not sit in the chair if it was in room B. Thus, we could summarize a state that is not possible as:

R: {B}, robot is in room B (which does not have the TV and chair in it),
D: {O}, door is open (so robot can go room to room),
C: {P}, chair is occupied
T: {N}, TV is "ON"

We can characterize the above state as not being *logically possible*. In other words, the above state violates certain common-sense system constraints. For example, one constraint requires that the robot must first be in room A in order to turn on TV. Another constraint requires that the door must be open in order for the robot to go from one room to the other.

System constraints can exert their influence along the dimension of time as well. This characteristic results when we consider a series of states. For example, lets consider just two states that occur in succession. Let's begin with a state:

R: {B}, robot is in room B
D: {C}, door is closed (so robot cannot go room to room A),
C: {E}, chair is empty
T: {F}, TV is "OFF"

In this case, the state of room A being occupied by the robot, R=A, is impossible because the door is in the state of being closed, D=C. Thus, the following two states could not occur in immediate succession:

Initial State	Operator	Next State
R = B →	CR →	R = A
D = C →	{No Operator}	D = C
C = E →	CC →	C = P
T = F →	CR →	T = N

1.5 State Space

From these examples, the following definitions are demonstrated. We have a *combinatorial space,* which is the set of all possibilities that result from all possible arrangements of the system's variables. Within the combinatorial space is a subset of states called the *state space,* which result from the system's constraints. The distinguishing characteristic that identifies the state space is that it is made up of those states that are *reachable*. Reachable states are states that the system **can** exist in after it transitions from the current state to the next state. The states that would be un-reachable then, would be those states that are logically impossible for the system to transition into. We will demonstrate below that the subset of states that comprise the state space will shrink in size as the system evolves.

For instance, refer to the example above: with a start state of R=B (the Robot in Room B), the next state cannot be: TV turned ON. This state is not reachable due to the system first requiring that the state of the door being changed first, which allows the Robot to change rooms and then turn ON the TV. Thus, the elements of the state space set are constantly changing depending on the sequence of operators and states.

1.6 State Space Table

Another useful representation of the state space is referred to as a *state space table*. Refer to Table 1.1 below. In this table we are doing several things simultaneously. First, we enumerate the set of states that represent all possible combinations of state variables. In the RTS we have four variables, each capable of being in only one of two states. This results in a total of 2^4=16 possible combinations. From these sixteen possibilities, only twelve exist within the state space due to the unique constraints that characterize the RTS. Also, in Table 1 we use an arbitrarily chosen initial state of BCEF. From this initial state we indicate whether or not each of the states in the state space are reachable and the minimum number steps required.

Table 1.1, Robot-TV State Space Table with Initial State of: BCEF

Variables					Number of
R	D	C	T	Reachable	Steps
A	O	P	N	yes	4
A	O	P	F	yes	3
A	O	E	N	yes	3
A	O	E	F	yes	2
A	C	P	N	yes	3
A	C	P	F	yes	2
A	C	E	N	yes	2
A	C	E	F	yes	1
[B	O	P	N]	no	
[B	O	P	F]	no	
B	O	E	N	yes	1
B	O	E	F	yes	2
[B	C	P	N]	no	
[B	C	P	F]	no	
B	C	E	N	yes	1
B	C	E	F	yes	0

[ ]: these states do not exist within state space, but are included in the table for the sake of indicating all combinatorial possibilities.

1.7 State Transition Table

The above table can be expanded to enumerate all initial states and all final states that exist for the system. Such a table is referred to as the *state transition table*. The transition table is essentially a matrix that lists all of the initial states across the top row and all of the end states down the column on the left. Within each square below each of the initial states is the necessary operator to achieve the corresponding end state. One key feature of the transition table is that it enumerates all end states that can be reached with only one transition. This situation can be summarized with the following schematic:

start state ---> one operator/transition ---> end state

1.8 State Transition Diagram

To graphically represent this set of transitions, a state transition diagram is constructed that has each state as a *node* and each transition as an *arc*. With these elements we can depict all of start state nodes, all of the "single step" transition arcs and all of the resulting end state nodes as a web of interconnected parts. For each of the arcs, the operator that corresponds to the particular transition could be indicated. Also, the arcs could have arrows on each end, and are considered to be *directed*. The arrow indicates which node corresponds to the start and end state for each transition. Refer to **Figure 1.2.**

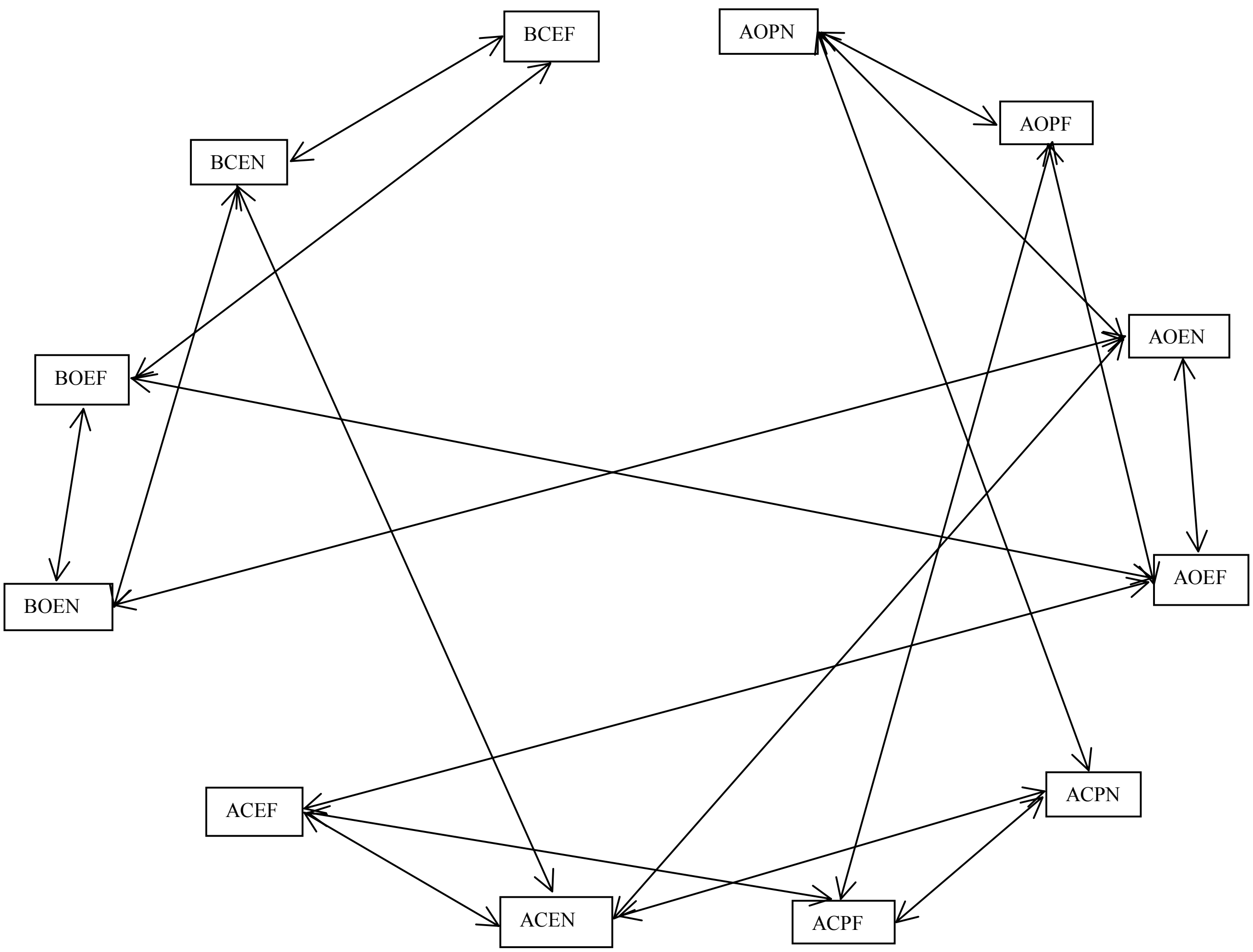


Figure 1.2, State Transition Diagram for RTS

1.9 Macro Transition Table: More about System Constraints, Possibilities and Reach-ability

One of the limitations of both the state transition table and diagram is that it does not include all of possible transitions. What is missing are all of the end states that require more than one transition. To construct an all-inclusive and "exhaustive" enumeration of all possibilities, we add all of the multi-step transitions into the body of the state transition table. This is accomplished by adding the corresponding operators into each cell of the table. Such a table, representing all of initial states, all of the necessary operators, and all the resulting end states, is referred to as the *transitive closure macro table* of the basic state transition table for the RTS. **Table 1.2**, below, allows us to observe some key characteristics of our RTS.

Table 1.2, Macro Table Representing Transitive Closure of the Basic State Transition Table for the RTS

	AOPN	AOPF	AOEN	AOEF	ACPN	ACPF	ACEN	ACEF	BOPN	BOPF	BOEN	BOEF	BCPN	BCPF	BCEN	BCEF
AOPN	Ǿ	CT	CC	CC CT	CD	CD CT	CC CD	CC CD CT	X	X	CC CR	CC CR CT	X	X	CC CD CR	CC CD CR CT
AOPF	CT	Ǿ	CC CT	CC	CD CT	CD	CC CD CT	CC CD	X	X	CC CR CT	CC CR	X	X	CC CD CR CT	CC CD CR
AOEN	CC	CC CT	Ǿ	CT	CC CD	CC CD CT	CD	CD CT	X	X	CR	CR CT	X	X	CD CR	CD CR CT
AOEF	CC, CT	CC	CT	Ǿ	CC CD CT	CC CD	CD CT	CD	X	X	CR CT	CR	X	X	CR CT	CD CR
ACPN	CD	CD CT	CC CD	CC CD CT	Ǿ	CT	CC	CC CT	X	X	CC CD CR	CC CD CR CT	X	X	CC CR	CC CR CT
ACPF	CD, CT	CD	CC CD CT	CC CD	CT	Ǿ	CC CT	CC	X	X	CC CD CR CT	CC CD CR	X	X	CC CR CT	CC CR
ACEN	CD, CC	CC CD CT	CD	CD CT	CC	CC CT	Ǿ	CT	X	X	CD CR	CD CR CT	X	X	CR	CR CT
ACEF	CC, CD, CT	CC CD	CD CT	CD	CC CT	CC	CT	Ǿ	X	X	CD CR CT	CD CR	X	X	CR CT	CR
BOPN	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
BOPF	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
BOEN	CC CR	CC CT CR	CR	CR CT	CC CD CR	CC CD CR CT	CD CR	CD CR CT	X	X	Ǿ	CT	X	X	CD	CD CT
BOEF	CC CT CR	CC CR	CR CT	CR	CC CD CR CT	CC CD CR	CD CR CT	CD CR	X	X	CT	Ǿ	X	X	CD CT	CD
BCPN	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
BCPF	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
BCEN	CC CD CR	CC CD CT CR	CD CR	CR CT	CC CR	CC CR CT	CR	CR CT	X	X	CD	CD CT	X	X	Ǿ	CT
BCEF	CC CD CR CT	CC CD CR	CD CR CT	CD CR	CC CR CT	CC CR	CR CT	CR	X	X	CD CT	CD	X	X	CT	Ǿ

With the macro table we can determine much about the system's state and state transitions. For example, given that the system has four variables, its start state is determined if we know the state of each of the variables at the beginning of a sequence of transitions. Also, if we are given a sequence of operators, we could then determine the final state. Or, equivalently, given a required final state, we could determine the necessary sequence of operators to achieve the end state.

Since, we know that with a set of four state variables, each capable of a maximum of only two states, we have a total of sixteen possible states. Within this set of sixteen states however, only twelve comprise the state space (due to system constraints). Notice that within the macro transition table, the total possible transitions is $16^2 = 256$. However, of the 256 possible state transitions, some are not reachable. The un-reachable states are the cells that are X'ed in the macro table. If you count them, there are 112 X's. Note also, that there are 16 cells on the diagonal that runs from the top left corner of the macro table, to the bottom right corner. The symbol in these cells, **Ø,** means "do not change". Obviously, these are the cells that correspond to the start state equaling the end state. So, from the original 256 state transitions possible in the macro table, only 132 = 256 – (112 + 12) actually apply to the RTS.

Further, the system constraints not only limit the size of the state space, but they also impose constraints on state reachability due to the logically possible sequence of operators. For example, we know that for the robot to go from say, room B to room A, the door must first be open. Thus, if the door was closed, and the robot had to go from B to A, then the change door operator would need to precede the change room operator. From this knowledge about the system's constraints, we know that in order to reach the desired end state, the operators must occur in the order indicated in the schematic below. Note that the order refers to an immediate and unidirectional flow.

CD ---> CR ---> CT ---> CC

Later, we will examine the impact of a bi-directional flow that allows for flow in both directions, as depicted in the schematic below.

CD <--> CR <--> CT <--> CC

1.10 Prediction and Probabilities

A convenient method of communicating predictions about system behavior is to express the chance of a certain state occurring as a *probability*. Probabilities are numerical values that can range from {0..1}. These values express our belief for the likelihood of a certain outcome – which is the state of the system. The closer the probability of a certain state occurring is to 1, then the greater our belief that is will actually happen. In the case of the RTS, we want to compare the state probabilities of the unconstrained system to the constrained system.

If you had to predict the current state of the RTS and it had no constraints, then you would have to make a choice from 16 possibilities (assuming that you also had no other information about the system state). This means that you would a have 1 in 16 chance of just guessing the correct state.

the probability of just guessing correct state for the unconstrained system = 1/16

Now, if the RTS has its constraints applied, then there are only 12 states in the state space that you would have to guess from (again, possibilities (assuming that you also had no other information about the system state). Now, you would a 1 in 12 chance of just guessing the correct state.

the probability of just guessing correct state for the constrained system = 1/12

This means that your chances of just guessing the correct state are a little bit better with the constrained system than with the unconstrained system because ***1/12 > 1/16.***

1.10 Reversibility and Irreversibility

Consider the following case illustrating hybrid directionality represented in the schematic below:

CD --> CR --> CT <--> CC

In this specific scenario, once the door changes its state from the start state, it does not matter if it changes again - a sort of one-shot-deal since the robot will change rooms only once. If the robot begins in room B, there is a good chance that the desired end state can be reached once the robot changes into Room A. However, if the robot were to begin in room B and the door changes state, to being closed, then the desired end state becomes unreachable. Then the remaining operators CR, CT and CC would not have any impact on changing the state of the system.

But, once the robot is in Room A, then the CT and CC operators can be executed and the end state can be reached. Further, the possibility of the operators CC and CT repeating in an alternating pattern is reachable. This possibility is allowable because the relationship between the operator that changes the state of the chair (from EMPTY to OCCUPIED) and the operator that changes the state of the TV (from OFF to ON) are independent of each other.

These scenarios demonstrate what is termed as *reversibility* and *irreversibility*. The term reversibility is associated with bi-directional operators, where "what was done can be undone", so to speak. Conversely, irreversibility refers to the unidirectional nature of certain operators because they will only "go one way". The uni-directionality of operators plays a very significant role in the behavior of systems - especially if the operator is a function of time. Also, note that directionality is very much tied to the nature of the relationship between the operators. Those operators that are dependant on other operators for their reach-ability (i.e., CR and CD) will tend to have unidirectional flows. Bi-directionality is often associated with operators that are independent to each other.

1.11 Abstraction

To continue exploring the possibilities latent within the RTS, we can implement the process of abstraction (which, by the way, is a form of operator itself). The process of abstraction allows us to remove certain attributes of the system such that the resulting "stripped down" system represents the set of attributes that uniquely characterize the system. For example, let’s say you have no idea what an umbrella is. So, I show you my umbrella:

> *It has a "J" shaped wooden handle and a blue nylon canopy that furls up around a wooden dowel about four feet long. It is about 3.5 feet in diameter when opened. Ok, now somebody else comes along and has one of those things that can fold up into a sort of cylinder about 1.5 feet long. It has a bicycle grip type handle. When a button on the handle is pressed, a disk of thin black cloth unfolds on a silver “shaft” that extends like a telescoping radio antenna.*

OK, I told you that what I had is an umbrella. Given the characteristics of my umbrella, you now surmise that the other object is also an umbrella. Even though these two forms of umbrella are different at the level of appearance, they are quite similar at the level of function – they keep the rain off of your head. This fundamental similarity, is the point of departure that allows us to describe umbrellas in a non-material and abstract manner that focuses on the characteristics that uniquely differentiate umbrellas from other objects. Simultaneously, these abstract qualities of umbrellas allow the possibility of blocking not only the rain, but also the sun, as is the case in a closely related object known as a parasol.

To apply the process of abstraction to the RTS we first must decide what are the essential characteristics of the system. To begin, we must choose the set of characteristics that determine the behavior and/or functioning of the system. For example, the RTS has four variables, twelve reachable states and four operators. It also has a set of constraints that govern the nature of the interaction between the various states. The process of abstraction for the RTS could begin by giving each of the states more generic names - such as a number from the set {1..12}. Next, lets give the operators more generic names also - such as a member from the set {A..D}. Let’s strip away the details of what each of the states and operators represents and just keep the nature of the interaction. For example, instead of thinking about the CD operator indicating the state of the door, lets just think of the operator governing any kind of binary state (i.e., on/off, up/down, left/right, etc). The same can apply to the remaining three operators. The process of abstraction has additional properties:

1) Choice of which details to remove. Deciding what is essential vs. unessential, depends on what you want to do. Relevant details are relative to observer's chosen point of view, usually utilitarian: to perform some useful function.
2) Abstraction reduces the system to variables, states and operators (or functions: such as determining the cost of changing from one state to another).
3) What remains is the relationships (i.e., operators) between each of states.

1.12 Functions: The Notation of Abstraction.

To expand the notion of abstraction, we are required to develop some unit of organization, or building block or *primitive*, much the same way that chemistry utilizes elements, biology utilizes cells, and so on. For our discussion we will use the variable as our fundamental unit of organization. Essentially, we can view a system, as a collection of components, which we think, may be significant.

The groups of elements can be represented as an *alphabet* of variables. Each group of variables can also be denoted as a set. The set of variables that describe the internal state of the system are called to *state variables*, as mentioned above.

Also, recall that state variables can undergo a transition from one state to another - from the initial state to the next state. The rule that describes how each variable can change is referred to as the *transition rule*. The set of all transition rules is collectively referred to as an *operator*.

External to the system boundaries are the *input* and *output* variables. The input variables are those observable elements that influence how the operator will dictate which "next" state the system will evolve to from the initial state. Once the system has arrived at its "next" state, it can produce a set of observable characteristics that are referred to as outputs. Note, that the end state refers to the values of the state variables which are inside the system boundaries and that the input and output variables are external to the system boundaries.

To express these aspects of a system in succinct terms, function notation is employed. A function is mathematical shorthand for describing how each element in a set can be converted into one, and only one, element of another set. For example, consider the following two sets:

Set #1 (X)	Set #2 (Y)
-2	-4
-1	-2
0	0
1	2
2	4

What's going on here? Set #1 is somehow converted into Set #2. After a moment's examination, the relationship between the sets is clear: multiply the first set by 2 and you get the second set. Using function notation, this relationship can be written as:

$$Y = f(X) = 2*X \tag{1}$$

What this says is that: "you take X, multiply it by 2 and you get some value Y". Expression (1) can also be written as:

$$X \longrightarrow f{:}2*X \longrightarrow Y \tag{2}$$

This expression is just another way of saying the thing, but represents it with arrows. The important meaning of the arrows in expression *(2)* is that: *X* could be viewed as an input, the function *f* as an operator and *Y* as the output. Expression (3) is the more general form of (2).

$$Input \longrightarrow f \longrightarrow output \tag{3}$$

To use some more function notation, we can use the elements from the Sets #1 and #2 above. In this format we express the function as an *ordered pair*. With the ordered pair notation, the value on the left and the value on the right always belong to sets #1 and #2, respectively. Written in a generic format, the ordered pair function notation looks like:

$$f(X, Y), \text{ or similarly,} \tag{4}$$

$$f(\,input, output) \tag{5}$$

So far, we have taken the view that the function expresses those variables that are external to the system boundaries: the inputs and outputs. Recall that the function can also express those variables internal to the system boundary – the state variables. In this case, the function expresses how a variable changes from one state to another. Refer to (6), (7), (8) below.

$$f(\,initial\ state\ of\ z,\ next\ state\ of\ z) \tag{6}$$

$$f(z, z') \tag{7}$$

$$f{:}\ z \longrightarrow z' \tag{8}$$

Note also, that in expressions *(6), (7), (8),* z could also represent a group of state variables. For example, z could represent all four state variables of the RTS. An example of this notation could be:

$BOEM \longrightarrow f \longrightarrow AOEN$, where f is the operator CR (change room).

Thus, with function notation there are at least two views of a system that are made explicit:

i) the variables as inputs and outputs, or
ii) the variables as initial and next states.

Both of these views are useful, depending on what the system is intended to accomplish. One example of a system described as a set of state variables is described below.

1.13 Black Boxes and Variables

Another way of thinking about variables and functions is called a *black box.* The idea here is that what goes on inside the box is not visible and therefore unknown to the observer. All we know is what goes in (the inputs) and what comes out (the outputs). The black box is another name for variable. The mechanism that transforms inputs into outputs would then be the function and/or operators. That the box is black is what obscures our ability to "look inside". It is usually true that if we want greater predictive power for a given system then we might need to analyze each black box to discover the function and/or operators that transform the inputs into outputs. The process of analysis would require the discovery of the black boxes that constitute the original black box. This recursive process of discovering greater and greater detail thus renders what was originally opaque to become transparent.

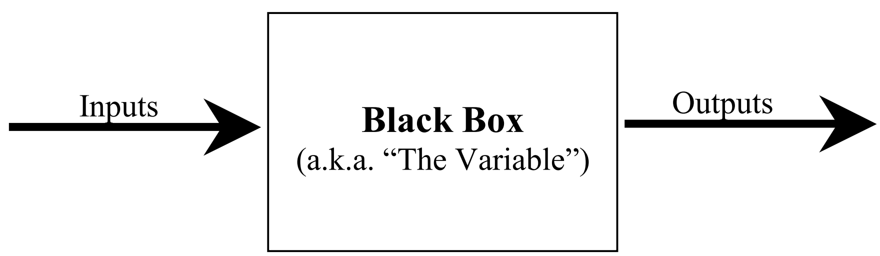


Figure 1.3, Generic Black Box

1.13 Finite State Machine

Once the system is abstracted, we can develop a logically valid version of the system that is called a *finite state machine* (FSM). As the name implies, such a system has a limited set of possible states. FSM's have some properties that make them slightly different from what we have dealt with so far. One significant difference is that the operators take on a slightly different role. For FSMs, the input symbols take on the role of operators. The operator can be thought of as being external to the system, and acts as an external force to impact the system's behavioral outcome. Additionally, since operators can be represented as a series of letters, these inputs can be "fed" into the system as a letter string. The end state is then observed. In particular, we might be interested if the end state is reachable or not.

Letter strings that achieve reachable states might serve some purpose, while those that achieve unreachable states are deemed useless. For example, when a computer program is written in some language, such as C, the human written code must be translated into computer readable code. This process is automated. The computer accomplishes this with another program called a *compiler*. The compiler takes each piece of human written code and translates it into computer readable code. However, how does the compiler determine if a fragment of human written code is syntactically correct?

This is where the FSM comes in. Each piece of written code is fed into the FSM as an operator. Each "phrase" of code comprises a set of operator "letters" which form strings that act as inputs into the FSM. The important part is whether or not the inputs, in the form of operators, result in a final state for the FSM that is referred to as an *accepting state*. If an accepting state is obtained, then the code will at least be readable by the compiler. If an unacceptable state is encountered, then the compiler will detect a particular form of error referred to as a *syntax error*. Note that the compiler cannot determine the validity of the higher level logic governing the overall purpose of human written program itself. The compiler deals strictly with the syntax of each written code phrase.

For example, let's use an English language sentence: "The robot is sitting in a chair that is empty ". The sentence is grammatically correct. The subject, verb, and object components are arranged and structured correctly. This is analogous to what the compiler determines. However, in the highly constrained context of the RTS, the higher level logic behind the actual meaning of the sentence is not correct. Recall that in the RTS, the state of chair is Occupied when the Robot sits in it – thus the state of the chair is logically inconsistent with RTS if its state is Empty when the Robot is sitting in it. A sentence that is both syntactically and logically correct, would be "the robot is sitting in a chair that is occupied". In the case of present day compilers, these types of logic errors cannot be detected.

As discussed above, we went through the process of abstracting the system. The end result is that we now have a set of variables, states and operators that are generically named. In other words, what we have left after the RTS is abstracted, is the "essence" of the system. What we can do with this essence is to use it to better understand other systems that may differ in the details yet share the same essence. Let's demonstrate this with the following example.

1.14 Isomorphism

Consider a system called "the adult-proof kitchen stove system", or just, the stove system (SS). With the SS we have a single burner that is controlled by the position of two knobs. For this example, we will not concern ourselves with the state of the burner as a result of adjusting the knobs. Instead, we will focus just on the position, or states, of the two knobs. On the stove console, one switch has a triangular knob. To the right of triangular knob is a switch with a square knob. Refer to **Figure 1.4**. Note how the two switches are labeled.

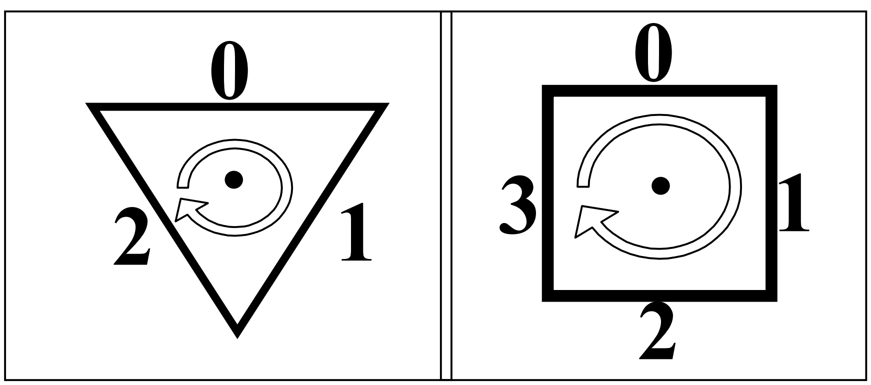


Figure 1.4, Adult-proof Stove Switches.

For the states of each switch, we will use the upper knob label to identify the state. The left knob (called "T", for triangle) has three settings: {0, 1, 2}. The right knob (called "S" for square) has four settings: {0, 1, 2, 3}. Thus, for the SS we have the following components so far:

Variables	States (based on upper knob setting)
T	0, 1, 2
S	0, 1, 2, 3

Now that the variables and states are known, we can make explicit the state space.

T	S
0	0
0	1
0	2
0	3
1	0
1	1
1	2
1	3
2	0
2	1
2	2
2	3

Another way of expressing the state space is to use set notation. This results in a collection of 12 ordered pairs of numbers, each pair representing a single state of SS. The pairs are ordered because we consistently use the right-to-left order: {T, S}.

(0, 0), (0, 1), (0, 2), (0, 3),
(1, 0), (1, 1), (1, 2), (1, 3),
(2, 0), (2, 1), (2, 2), (2, 3)

The operators for the SS are based on <u>the number of clock-wise "clicks" for each knob</u>. Each click changes the upper knob setting by an increment of only one:

T1 = one click to the right for only the T-knob
T2 = two clicks to the right for only the T-knob
S1 = one click to the right for only the S-knob
S2 = two clicks to the right for only the S-knob

These four operators can be arranged into various combinations. For the purposes of this example, it is sufficient that the SS have only non-repeating series of operators. So, using set notation again, we have:

(No Operator), (T1), (S1), (T2), (S2), (T1,S1), (T1,S2), (T2,S1), (T2,S2)

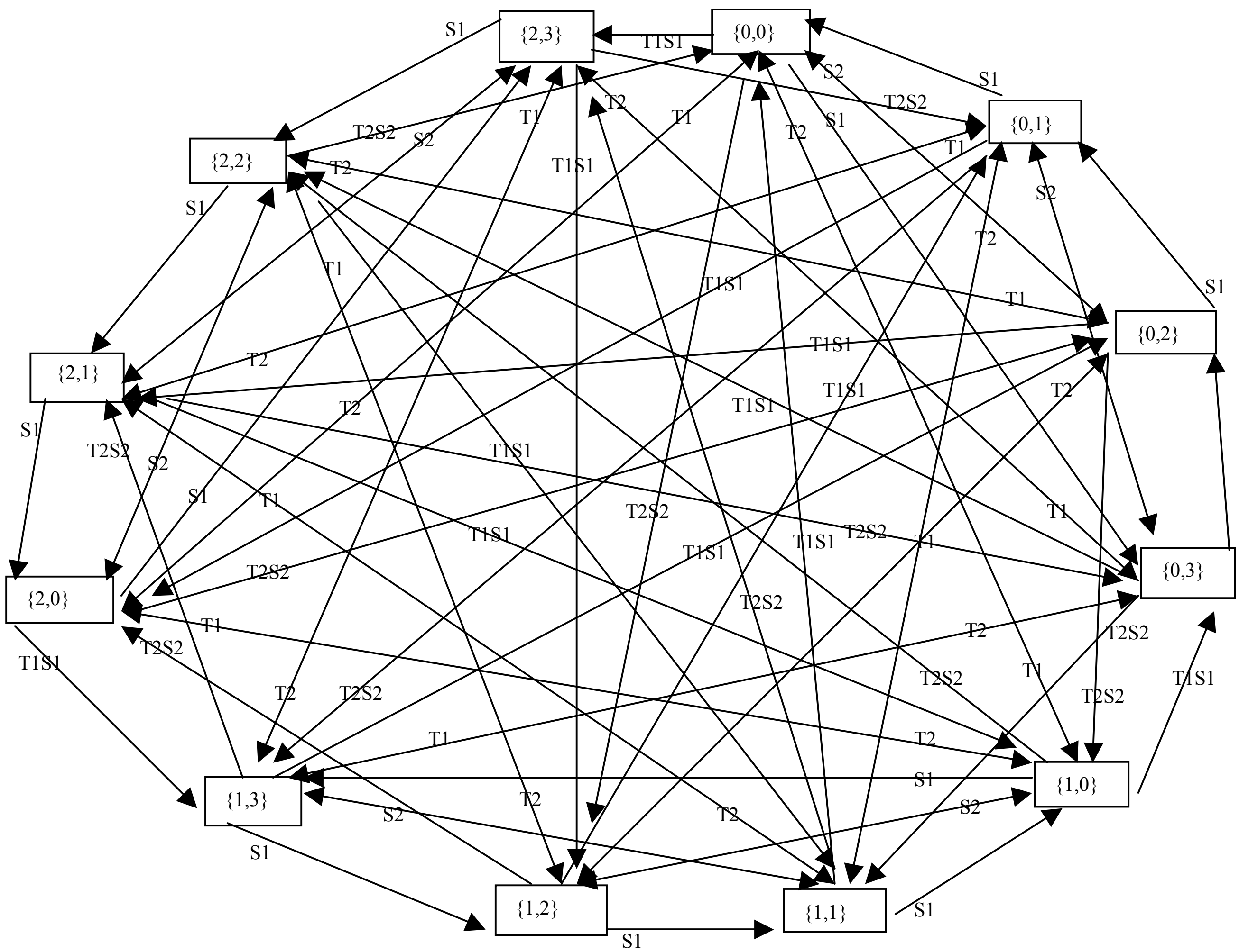
With the states and operators, we can now construct the state transition table. Recall that the state transition table begins with one state, applies only one operator, and then ends with an end state. For this particular transition table we will apply the T1S1 operator to each initial state. Note that we could apply any of the above operator sets to SS.

Initial State	Operator	End State
0 0	T1S1	1 1
0 1	T1S1	1 2
0 2	T1S1	1 3
0 3	T1S1	1 0
1 0	T1S1	2 1
1 1	T1S1	2 2
1 2	T1S1	2 3
1 3	T1S1	2 0
2 0	T1S1	0 1
2 1	T1S1	0 2
2 2	T1S1	0 3
2 3	T1S1	0 0

From the state transition table we can construct the state transition diagram. As before with the RTS, we take each state and represent it as a node. We then apply each operator to each state-node. The application of an operator to a node is represented as an arc. The operator-arc then connects each initial state to the end state. Refer to **Figure 1.5** for a nearly completed graph.

Exercise:

Identify the missing arcs in Figure 4 (hint check out the T2S2 operators…).



<u>**Figure 1.5**, State Transition Diagram for Stove System.</u>

Now, can you recall why we undertook this exercise in the first place? The point here is to realize that the RTS and SS have some very interesting commonalties. First, let's compare their state space tables. Refer to Table 1.3.

<u>**Table 1.3**: State Space Tables for RTS and SS</u>

States for RTS	States for SS
A O P N	0 0
A O P F	0 1
A O E N	0 2
A O E F	0 3
A C P N	1 0

A	C	P	F	1 1
A	C	E	F	1 2
A	C	E	F	1 3
B	O	E	N	2 0
B	O	E	F	2 1
B	C	E	N	2 2
B	C	E	F	2 3

Notice that these two tables are very similar: each is a list of 12 items, each item representing a unique state. Further, now compare the state transition diagrams for the two systems. Refer to Figures 2 and 4. Again, notice that these two graphs are quite different. Notice that what is different is not the number of states, but instead the number of "allowable" transitions. The RTS has a greater number of constraints than the SS.

Exercise:

Determine the set of constraints that would result in the SS to have a transition graph to be exactly the same as the RTS transition graph.

If the two transition tables and graphs were identical, then the systems can be referred to as *isomorphic*. The transition table and graph represent core components of each system's essence. The property of isomorphism between separate systems allows us the potential to transfer insights known about one system to other, less understood system. Generally, this is a good idea because it is a faster and relatively less expensive way to learn about systems.

1.15 Isomorphism and Mapping

Returning to function notation, isomorphism manifests as a special case where the two sets being related have the same number of elements, or *cardinality*. We can observe this by comparing the sets of ordered pairs. When the two sets have the same cardinality, they are said to *map* onto each other. Mapping can imply that two items translate directly onto each other. In the case described above, the two isomorphic systems could map onto each other. This is the strong version of mapping. In its weaker form, mapping can connote simply that two items are related but slightly different.

An example of the strong form of mapping would be the ordered pairs of states found in a state transition table. Recall that in a state transition table, we have a one step transition from the start state to the end state. So, if we further abstract the RTS and the SS, we could represent the state for both of these systems as shown in **Table 1.4**:

Table 1.4: State Space Tables for RTS and SS

State	States for RTS	States for SS
a	A O P N	0 0
b	A O P F	0 1
c	A O E N	0 2
d	A O E F	0 3
e	A C P N	1 0
f	A C P F	1 1
g	A C E F	1 2
h	A C E F	1 3
i	B O E N	2 0
j	B O E F	2 1
k	B C E N	2 2
l	B C E F	2 3

Once we have given the states identical names we can then list the possible state transitions as order pairs. Given the assumption that the two systems are isomorphic, then they would be mappable. Refer **Table 1.5.**

Table 1.5: State Transitions for Mappable Version RTS/SS

(no transition)
(a,a) (a,b) (a,c) (a,e)
(b,d) (b,f)
(c,d) (c,g) (c,i)
(d,h) (d,j)
(e,f) (e,g)
(f,h)
(g,h) (g,k)
(i,j) (i,k)
(j,l)
(k,l)

Chapter 2

Characteristics and Laws of Systems

Introduction: Diversity and Symmetry

From the RTS discussed above, we noted that there are a finite number of observed states that the system can exist in. That there are a finite number of observable states is often an artifact of the observer and not necessarily a distinguishing characteristic of the observed system itself. The states for both the RTS and SS are *discrete*, meaning that there is no "gray zones" between each of the states - the system is in either one state of the other. However, there are systems where the variable states are *continuous*. For example, a variable for a system could be temperature. The temperature could be measured in degrees Centigrade. What is it that separates one temperature state from another? The short answer is that the finiteness of observable states results from the observer's precision in measuring the continuous system variable. *The more precise the measurements, the more potential states each variable can obtain from the standpoint of the observer.*

We are concerned with the number of states the system can be observed in because we wish to know what parts of the system change and which parts do not change as the system evolves. At this juncture we can now introduce the concepts of *diversity* and *symmetry*. To begin, just think of diversity as those parts of the system that can change and the parts that can not change as symmetry. The more states a system can exist in, the greater its diversity. This is where the state space comes into the picture because the state space makes explicit those states that the system can exist in. We will return to this subject again. However, for now there are some additional preliminaries that need to be covered…

2.1 System Boundaries: Closed to the Rest of the Universe

Basically, the intuition behind the concept of closed-ness hinges on the notion of boundaries. Once a set of variables has been defined for a system, then the system boundaries have also been defined. The importance of these chosen variables is that they are the set of factors that, when known, can be used to determine the state of the observed system. When this set of variables has been obtained, we can think of the variable set as being closed - in other words we have a sufficient set of variables to know our observed system. A closed system then will not have any "hidden" variables that exist in some other system that impact the observed system. If a variable outside the system does have an impact, then it must be included in the set of system variables. Once we are convinced that we have a closed variable set, or closed system, then we can make other observations of our system.

Symmetry and diversity are one set of characteristics that systems exhibit. When we think about such concepts as diversity and symmetry, we must always keep in mind that we are dealing with a special class of systems: those that are closed. Without meeting this necessary condition, the following discussion has less, but still significant value. Sometimes it is necessary to deal with systems that are partially closed (i.e., system has some variables that are too expensive to measure). So, what we find is that sometimes there are degrees of closed-ness. In which case, the following discussion will be applicable to the degree that the observed system is closed.

In the RTS, we noted that there are a total of 16 possible states. We determined the number of states by knowing that there are four variables, and that each variable can exist in just one of two possible states. So, if you have m variables and each variable has the same number of N possible states then there are N^m combinations of state variables. You can think of the combinations as forming a set combinatorial of possible of variable states.

It follows then, that for the RTS there is the set of states represented by the *combinatorial space* that numbered $16 = (2^4)$. Then, after we eliminated the states that are logically not possible, what remained is the set of states that comprise the state space of 12 possibilities (4 of the original 16 are not logically possible). What is significant here is that from 16 combinatorial possibilities, four are eliminated. This means that the system has lost one quarter of the total number of possible states. This is significant because of several reasons.

For example, let's say that we are studying a very simple system that involves a single coin that is flipped. After being flipped, the coin system can only obtain two states: heads or tails. For our purposes, we want to be able to build a model that will predict what end state the coin system will obtain after it is flipped and has stopped moving. Initially, we might guess that the system has a 50% chance of ending up heads or tails. We arrived at this guess because the combinatorial state space and logical state spaces could map onto each other, resulting in a set composed of only two possible end states: heads or tails. Since the observer knows nothing about the system, we assume that each state is equally possible: two states, equally possible, 100% divided by two states = 50% chance of each end state occurring. This reasoning sounds feasible enough.

Now, let's apply this reasoning to the RTS. Suppose that we want to be able to guess what the present, or next state will be at any given time. Also, lets assume that we are observing the RTS for the first time, and all we know is the following:

* that there are four variables and
* that each variable can only be in one of two states, and
* that we know the current state.

At first we might be able to only observe the system changing from one state to the next state. Well, if we are just guessing, the question that arises is: what are our chances of guessing the correct state? In other words, what are our odds of guessing correctly? If we do not know anything about any of the logically impossible states, then we could resort to calculating our odds of guessing correctly based just on what we know about the combinatorial state space. What we do know is that there are four variables, each capable of two states, totaling 16 (= 2^4) states. To calculate the odds of guessing the correct state in the combinatorial state space table, we have the 16 possibilities. This represents the system without any constraints. This means that to just guess which state will result from any initial state, we would have a 1 in 16 chance of guessing correctly. Next, let's apply the constraints to the system.

As mentioned above, with constraints, the state space table has the number of possibilities reduced from 16 to 12 due to 4 states that are eliminated. So, now that 4 possibilities are eliminated, our odds of correctly guessing the next state went from 1 in 16 to 1 in 12. So, what this demonstrates is that the constraints placed on a system by characteristics such as logical impossibilities increases the odds of just guessing the correct state. In other words, constraints reduce the number possible states a system can obtain and can thus increase the odds of correctly predicting system behavior.

Diversity is the term used to describe the number of different states a system can obtain - this is the combinatorial space. Symmetry refers to the set of constraints that can reduce the set of reachable states that originally existed in the combinatorial space. This resulting subset of reachable states comprises the state space. These constraints can take any form. For the RTS, and many other systems, constraints will take the form of logical impossibilities. Most importantly, symmetry often occurs due to constraints that we can informally refer to organizational forces.

Just think of a germinating seed of grass. The growth of the seed is controlled by the genetically encoded "guidance system". This guidance system represents a prime example of an "organizational force" found in all biological systems. It is important to note that whatever the form, constraints reduce the number of possible states and produce forms that are patterned. Thus, the state space is almost always smaller than the combinatorial space. Also, note that as observers of a particular system, though we may not be able to recognize the underlying patterns latent in the system's symmetry, what appears as chaotic behavior may indeed be quite organized and patterned.

2.2 More on Symmetry and Diversity

The term symmetry always exists within the framework of how the observer is observing the observed system. This is a result of the variable set chosen by the observer to characterize the observed system. Further, the variable set was chosen by the observer with a specific point of view and purpose in mind - the intent of the model in the first place. The observed presence or lack of presence of symmetry in the system model is relative to the point of view and purpose of the observer.

2.3 Granularity or Scale

The *granularity* or *scale* of the observer's point of view will also influence the existence of diversity and symmetry for a given system. The notion of granularity focuses on the scale of observation and the fundamental units of observation utilized to measure each of the variables. The units of measurement can be degrees Fahrenheit temperature, grams of weight, number of individual cells or animals to make up a population size, etc. Granularity deals with the level of precision of the measurement. Are you weighing to the nearest ton? nearest ounce? nearest 10,000th of a gram? If you were doing a study on the weight of migrating barn swallows and you needed to sample individual weights - would you weigh them to the nearest kilogram? gram? or nearest 100th of a gram? The finer the measurement – the smaller the unit of measurement - the more precise the measurements. In this case, it might be most appropriate to weigh the birds to the nearest 10th of a gram. This might be an especially relevant issue if measurements are being taken out in the field. Attempting to weigh birds to the nearest 10,000th of a gram would be nearly impossible out in the field because a measuring instrument with that level of precision (i.e. the scale itself) would be so delicate that field usage might cause damage from just being carried, jostled about, possibly dropped, etc.

Granularity is relevant to this discussion because the amount of diversity and symmetry observed will be partially determined by the level of granularity. For instance, if we are observing the biochemical processes governing the development of a germinating seed of grass, what would we learn if we chose different units of measurement such as: individual electrons, or single elements (such as only carbon, hydrogen, oxygen and nitrogen), or molecules, or organelles, or cells? Depending on which unit of observation we chose to deal with, we might observe very different amounts of symmetry and diversity for the same system. Needless to say, the amount of symmetry observed with each unit of observation would be in some way related to amounts of symmetry observed with other units of observation - but they could still be quite different and lead to different sets of observations and conclusions about the same observed system.

2.4 What Changes and What Stays the same in Time and Space ?

Once an appropriate level of granularity or scale has been chosen and has been matched to the point of view and purpose of the model, symmetry will reveal those parts of the system that remain unchanged as the system evolves through its state space. What this implies is that there are certain system variables that change very little or not at all as the system changes with time. These variables are limited due to whatever forms of constraints are impacting the system.

Symmetry can also exist in space. Geometrical examples of spatial symmetry are common. Take the equilateral triangle. It is symmetrical about its geometric center as well as about the axis that bisects a side at right angles. The simple circle is totally symmetric about its geometric center from all angles. Another interesting example is demonstrated with morphing software that allows the combination and transformation of one image into another. Morphing the image of someone's face into a cat face that shares some of the features of the original face (such as eye color, mouth and nose shape, etc) is a form of symmetry. In this case symmetry manifests as the spatial patterns that result in recognizable features.

Information exists both in space and time and is also subject to the phenomena of symmetry and diversity. Communication lines such as telephone wires and computer network cables move information through space. Memory and data storage devices move information through time. The efficient transport of information through space and time will, in part, depend on the degree to which the transported information is NOT changed - or loses its symmetry between source and target. After all, if a network cable is somehow altering the signals passing through it, then the receiver of the signal might not be able to "understand" the message. Similarly, if a data storage device (i.e., RAM or hard drive) is not able to determine the location of each cylinder, or sector on the disk, then data will be lost - which is a dis-integration of symmetry.

2.5 Measuring Symmetry and Diversity

Already the comparisons between symmetry and diversity have been hinted at by the comparisons of the odds for guessing the system's state. When attempting to guess the state of RTS, the smaller the state space, the better the odds of guessing correctly. As mentioned above, this phenomenon results from the constraints inherent in the RTS. These constraints greatly impact the system's behavior by imposing a limited set of patterns. For the purposes of better understanding a given system, or understanding sets of different systems, it is often useful to derive some relative measure of diversity and symmetry. To accomplish this we can borrow some of the concepts developed in the field of information theory.

To begin, we need a way of encoding information in a way that is easily accomplished by some kind of machine. These days of course, the machine takes the form of a silicon based computer chip. Since the reason for creating the machine in first place is to do some kind of work, the cost of building the machine is a significant factor. The easier we can encode information with a machine, the lower the cost of the machine. The first step to encoding information cheaply is to have a VERY simple coding system. To be more specific, this encoding system is essentially a language. A language needs to have a set of "letters" that can form an alphabet that can be inexpensively arranged and, at the same time, allow for a rich vocabulary.

Evidently, it turns out that the least cost alphabet (based on today's current state-of-the-art) is achieved with just two letters, lets call them [0, 1]. We can arrange strings of these letters as long as we want in order to form words. In turn, these words can be arranged to form "sentences" which are used to encode information. This two-element alphabet is often referred to as a binary code. Each binary element in a word is called a binary unit or bit. For our purposes of characterizing systems, we can use binary code for both encoding the number of states that comprise the combinatorial and state spaces. What this boils down to then, is that we will be counting in base 2 instead of base 10. Remember that the significance of base 2 derives from the world of binary machines, which in turn have to be built as inexpensively as possible.

Next, we need this base two encoding system for labeling each of the states of the subject system being studied. For the RTS this means that we need to label or "tag" the combinatorial space of 16 states. So, we might use the following scheme:

RTS State	Base-two Label	Base-ten Equivalent
0000	0000	0
0001	0001	1
0010	0010	2
0011	0011	3
0100	0100	4
0101	0101	5
0110	0110	6
0111	0111	7
1000	1000	8
1001	1001	9
1010	1010	10
1011	1011	11
1100	1100	12
1101	1101	13
1110	1110	14
1111	1111	15

At this point you may be asking: HEY, WAIT A MINUTE!!!!

HOW COME THE RTS STATE IS EXACTLY THE SAME AS THE BASE-TWO CODE???

Well, the answer has two parts. First, is that the author has *arbitrarily* matched up each RTS state with a corresponding base-two label. As far as the RTS is concerned, it does not matter which label each state receives. The labels are for the benefit of the observer, not the observed. The second reason the RTS can have an identical base-two label is that the RTS has four variables that are each binary in nature - which is of course the same as the base-two encoded labeling system. So, we can generalize and say that: when system variables are binary, they can be mapped directly onto the base-two labeling system.

Next, lets look at the number of bits that are needed to label a set of sixteen states, or elements (counting from 0-15 allows us to label 16 elements). Upon inspection, we can count with four bits since it takes a four base-two number "places" to label all sixteen states. This base-two binary number (think: "word") is structured in exactly the same way as the base 10 numbers (think: "words"). Starting from the right-most digit we proceed left-ward with exponential increments:

1111
| | | |__= $(2^0)*1 = 1$
| | |___= $(2^1)*1 = 2$
| |____= $(2^2)*1 = 4$
|_____ = $(2^3)*1 = \underline{8}$
15

2.6 Bits

What we are most interested in however, is that it "costs" a four-place base-two number, or four bits, to count the first 16 integers (0..15). This cost can be expressed as *bits*, in this case, four bits. One of the uses of measuring bits, is that it allows us to compare the relative costs of diversity and symmetry within the same system as well as between different systems. We will discuss this point further in subsequent chapters.

Another aspect of the "cost" factor is related to the likelihood of guessing the correct state of a system. Referring yet again to the RTS, we know that the combinatorial state space is a set of 16 elements. The chance of just guessing the correct element is 1 in 16. Another approach is to have a computer search the combinatorial space. In the case of a program that is executing a search algorithm, the smaller the state space, the less time is required to find the state that corresponds to the system's actual state.

For the RTS we noted that the symmetry of the system reduced the set of states from 16 to 12. Thus, searching and finding one element from a set of 12 elements is easier than searching and finding one element from a set of 16 elements. So, in a sense, we can view the quantitative binary coding of set size as an index of "search space" magnitude. The smaller the search space, the "cheaper" and easier it is to find the correct element, or state.

2.7 From Base-Two to $\log_2$

OK, so now we have some idea of how to use base-two numbers for labeling each state and encoding the size of the combinatorial and state spaces. Recall that the combinatorial space refers to a system's diversity without constraints and that the state space refers to that subset of the combinatorial space to which the constraints do apply. Also, recall that our goal is to figure out the state of our target system at some point in time. What we want is a convenient and easy-to-understand method for comparing the magnitude of the combinatorial and state spaces. Also, we want to use the base-two-machine-encoded form of information that is measured in bits.

Well, the $\log_2$ function gets us this value very quickly. We simply take the size of combinatorial and state spaces and calculate the number of bits. For instance:

RTS Combinatorial Space = $\log_2(16) = 4.000$ bits
RTS State Space = $\log_2(12) = 3.585$ bits

We could also compare these two values in the form of a ratio:

$\log_2(16): \log_2(12) =$ 4 bits: 3.585 bits = 1.116

With this ratio of 4 : 3.585 = 1.116, we have a useful index that allows us to compare the RTS to other systems, or even to compare the RTS to itself over time to determine if the system's symmetry was staying the same or increasing.

This comparison allows us to formalize the manner in which we can quantify the magnitudes of difference between diversity and symmetry for a given system. In order for us to accomplish this, we rely on a very precise and quantifiable form of information - namely the base-two machine usable form.

2.8 The Fundamental Equation

From this comparison of the combinatorial and state spaces we now proceed to calculating the symmetry of the system. The intuition behind the derivation of symmetry is quite straightforward.

Basically, we know that the combinatorial space represents the total diversity of the states, or elements, that comprise the system. Next, we know that due to constraints, the combinatorial space is reduced in size to yield the state space. Thus, the symmetry of the system can be expressed as the difference between the combinatorial and state spaces. Recall that we are adopting the base-two machine encoding approach to information quantification, so we will express this relationship as:

$$\log_2(\text{system symmetry}) = \log_2(\text{combinatorial space}) - \log_2(\text{state space}) = \text{quantity of bits symmetry} \qquad [I]$$

For the RTS, this relationship would look like:

$$\log_2(\text{symmetry}) = \log_2(16) - \log_2(12) = 4 - 3.585 = 0.415 \text{ bits}$$

So, what the above relationship says is that there are 4 bits of information in the combinatorial space, and 3.585 bits of information in the state space. The difference is the quantity of information in the system symmetry, which is 0.415 bits.

Note that a convenient feature of log functions is demonstrated in the following equivalent expression of [I]:

log_2 (symmetry) = log_2 (combinatorial/state space) = log_2 (16/12) = log_2 (symmetry) = log_2 (1.333) = 0.415 bits

Another form of expression [I] is:

log_2 (combinatorial space) = log_2 (state space) + log_2 (system symmetry) *[II]*

This says basically the same thing as [I], but just has things arranged a little bit differently.

The ability to calculate the number of bits necessary to encode the diversity and symmetry that characterizes a system allows us to now compare a broad range of systems. **Figure 2.1** groups systems within the framework of two-dimensions: diversity and symmetry.

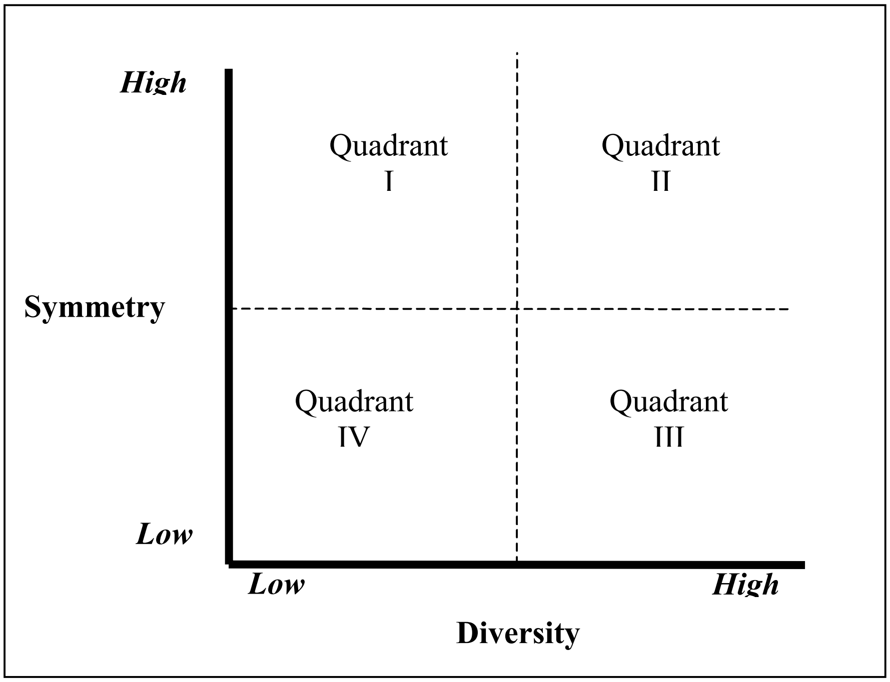


Figure 2.1, Graph of Comparative Systems

Quad I: low diversity, high symmetry: industrial plant
Quad II: high diversity, high symmetry: weather, stock market
Quad III: high diversity, low symmetry: chaotic systems, ie,war
Quad IV: low diversity, low symmetry: dice

Within the above two-dimensional framework of diversity and symmetry, we can classify various systems. Quadrant I characterizes those systems that are built to do one thing very well. Such systems would also require a considerable degree of complexity, i.e., many variables that are all operating within a very narrow range of states. The example given is an industrial plant, such as an oil refinery, or electrical power generating plant. These colossal machines are very complex, but do only one thing, hence the low diversity and high symmetry.

The Quadrant II systems exhibit the greatest amount of both diversity and symmetry. These systems are often referred to as *chaotic* - though their behavior appears random, there is an underlying order latent within both the "surface appearance" and subtler behavioral patterns. The weather and the stock market are just two examples of systems that are made up of millions of variables, each of which appears to have a multiplicity of possible states. Also, these systems present a great challenge to the modeler due to their vast complexity. Modeling such systems requires huge amounts of computational capability. The modeler selecting the most significant subset of variables can often resolve this challenge by choosing variable states that can satisfy the requirements for both accuracy and computational resource limitations.

When chance plays a very large role in the behavior of a system, owing to the random behavior of a majority of the variables and their states, we find systems belonging to Quadrant III. These systems are characterized by high diversity without the degree of symmetry exhibited by those systems in Quadrant II. The systems of Quadrant III are less influenced by their past. Their states tend to transform from one moment to the next in a more independent manner. However, keep in mind that these systems are still capable of patterns and tendencies. What we doing here is distinguishing systems along a continuum.

For Quadrant IV, we find systems that are limited in both their diversity and symmetry. Dice are an example due to their limited state space and the degree to which chance characterizes the outcome of each throw. This is made evident by the fact the outcome of each throw is completely independent of the previous throws - at least when the dice are fair...

At the limits of both diversity and symmetry we find systems that can change a lot to those that do not change at all. Complete diversity would characterize a system where each element operated completely independently of all the other system elements, independent of time, space and everything else. In point of fact, it is arguable whether such a collection of elements constitutes a system at all.

At the opposite extreme, we have absolute symmetry. Matter at absolute zero might exhibit complete symmetry. In this case, it is theorized that a uniform crystalline lattice structure develops where all atomic particles are positioned in stationary patterns that would epitomize symmetry. On the other end of the kinetic scale (i.e., extremely high temperatures), matter in a very high-energy plasma state also exhibits a high degree of symmetry - to a point. From the standpoint of matter density, we could find the plasma density quite uniform and homogeneous - which is a form of symmetry. However, from the standpoint of determining the presence of a spatial or geometric pattern existing between the particles - well, this would probably be impossible. Contrast this condition, with matter cooled off to near absolute zero, where atomic particles are neatly fitted into the lattice structure. Thus, depending on what you are modeling, whether it be spatial patterns of molecular geometry, or density, or information, diversity and symmetry will vary.

The above discussion has involved the notion of symmetry displayed by a system as the result of constraints. For example, the form of these constraints came as the results of rules given for the RTS (i.e., the logically unreachable state of the robot simultaneously sitting and a chair that is empty). However, when exploring an unknown system, the constraints are also necessarily unknown. And so, the curious and intrepid researcher must resort to a different set of tools for assembling a realistic model of the subject system. One of the most powerful uses of the fundamental equation is to view the symmetry of a system as the absence of diversity (diversity combinatorial space - diversity state space = symmetry).

2.9 Of Likelihood and Probability

The above discussion has touched upon the notion of likelihood – the likelihood of guessing the state of the subject system. The concept of likelihood is derived from the concept of probability. To illustrate how probability relates to our present discussion, we will embark on yet another thought experiment.

Imagine that you are given a pair of dice - the regular six sided kind, with one of the numbers 1..6 on each side. You are told that one of the die is rigged - lets say this means it comes up "one" a lot more than is expected. Next, you are told that the other die is fair. Then you are told that it is your job to determine the following:

* which die is rigged and which is fair,
* an estimate of "how rigged" the rigged die is, and finally,
* an estimate for how both die will behave in the future.

So to get started, the question now is: how does one go about analyzing the fairness, or lack thereof, of dice? Well, one way to go about this study of dice fairness is to set up a kind of experiment. First, we need to determine how many different states each of the die can exist in. Luckily, this is easy enough: each die has six sides and thus can only "show" a value of one through six, [1...6]. So, we have just six states to contend with. Next, our intuition tells us that we need to test each die by throwing it a lot of times to see how often each of the states shows up. OK, so let's say we have lots of time and thus will throw each die 1,000 times. Each throw will represent a sample of the die states after a throw. As we throw the dice, we record the result in the following manner:

Throw	Die #1	Die#2
1	3	1
2	1	1
3	5	6
4	4	1
5	2	1
6	6	1
.	.	.
.	.	.
.	.	.

So, now lets suppose that we have done all 1,000 throws for each of the die. Now we want to make some sense of the data we have collected. At this point what do you think we should do?

Well, one idea is to do some grouping. Let's count up the number of times, or *frequency,* each of the six states occurred for each of the die. Since there are 1,000 throws, we know that the total number of state occurrences should add up to 1,000. Imagine that the data set looks like this:

	Frequency for	
State	Die #1	Die #2
1	160	642
2	177	70
3	168	68
4	164	75
5	171	69
6	160	76
Total	1000	1000

Some patterns should jump out at you. The data set for Die#1 appear to indicate that the frequency of occurrence, or number of times each state occurred, are clustered around the 160-170 range. The data set for Die#2, shows that state #1 occurs almost ten times more than any of the other states for Die#2. Ok, intuitively, we can see that Die#2 is the rigged die. So, we have completed the first part of our goal: determining which die is loaded. But, now we have to estimate how much it is loaded. How in the world could we accomplish this?

One useful method is to express frequency of occurrence as a percentage. So, for instance, state #1 for Die#1 occurred 160 times in a total of 1,000 throws. This means that state#1 occurred 160/1000 = 0.16, or 16%, of the time. If we do the same calculation for each state for each of the die we get the following data, which is often referred to as a *frequency distribution:*

	Die #1	Die #2
State	% Frequency	%Frequency
1	16.0	64.2
2	17.7	7.0
3	16.8	6.8
4	16.4	7.5
5	17.1	6.9
6	16.0	7.6
Total	100%	100%

Hmmmm: Die#2 definitely looks rigged and Die#1 looks fair. But, in order to determine just how loaded Die#2 is, we first have to know what exactly a fair die is so that we can make the comparison between rigged and fair. So, what is a fair die?

An easy way to express the fairness of a die is to say that each of the sides has an equal chance of showing up. This means that there is a 1 in 6 chance of each side showing up, or a 1/6 = 0.167 or 16.7% chance of occurring. Notice, that if we say a die has, n, states then a fair die has a 1/n chance of occurring in any state. This condition of fairness in dice is closely related to what is also referred to as *random* - each state of the system has a 1/n chance of occurring, where n is the number of states in the state space.

So, now we can say that Die#1 is very close to being fair and random. Die #2 is in state #1 about 64% of the time. Normally, it should be in any state about 16.7% of the time, or roughly 64/16 = 4 times more likely to show up "1" than a fair die would. For the remaining sides, 2...6, Die #2 is less than half as likely to show them than a fair die (roughly 7/16=0.44 or 44%). In fact, using what is known as Bayes Rule, one could deuce how likely Die #2 is the unfair die, given the data.

OK, now to the last part of our thought experiment: we have to speculate about the future behavior of each of the dice. In order to accomplish this we will make a HUGE assumption which is to infer that what did happen in the past will tend to happen again in the future - that the behavior of each die will not change in the future. Thus, we rely on the statistical data (the frequency distributions) about our system to infer how the system will behave in the future. So, lets make the leap into assuming that our frequency distributions calculated from observations made in the past can serve as *expected frequency distributions* for events (i.e., states of the system) in the future. These expected frequencies give us a percentage value (i.e., like 16.7%) for the chance of a certain state occurring in the future. Another way of saying this is: there is a probability associated with each state occurring for each of the die.

Notice that these probabilities indicate that we are not certain about the outcome of any single event. This means that we cannot predict with 100% accuracy the outcomes of any particular dice throw. Probabilities just give us an estimate about the expected outcome of each dice throw. Also notice that, as expected, the fair die generates a frequency distribution that is close to random. This random distribution implies that each state has an equal chance of occurring (1/n).

[Footnote: Interestingly enough, there are statistical procedures that can provide a measure of how much "error" one could be expecting for each guess.]

Finally, note that when we begin to study an unknown system, we should also begin with the assumption that each of the states has an equal chance of occurring because we have no information with which to distinguish system states. We make this assumption on a provisional basis, realizing that the reality of the system's state space frequency distribution could be anything but random. At first, we do not know what this distribution looks like, and as we make observations about the system, we can then group the occurrences of each state into a frequency distribution to find patterns.

[Footnote: Truly, the notion of "fair" die is mostly academic since true randomness is both hard to achieve and prove.]

Now, to the relevance of probabilities to our discussion on calculating values for diversity and symmetry. The first piece to this puzzle is to note that, that the form of information that we are dealing with is content of a *signal.* This signal is a message that is communicated from sender to receiver. The content of the signal is whether or not a particular event has occurred. The signal has an information “value” as we have discussed above. The information value is measured in the form of bits. What is being measured is the probability of an event, or state, occurring. Events in turn comprise the state space of a particular system. The state space has an associated probability distribution. From the probability distribution we can estimate the degree of certainty that each state can occur.

The second piece to our puzzle is the following:

1) The more certain the occurrence of the event, the LESS the information content of the signal

2) The less certain the occurrence of the event, the GREATER the information content of the signal

The intuition here is that in the narrow sense that we are applying, a signal indicating the occurrence of a state with a small probability of occurring has more information content (i.e, bits) than a signal indication that a state with higher probability of occurring. We will return to this notion later on.

2.10 The Tale of Two Systems: Observer and Observed

Whenever we are studying a system, we are always faced with the interaction of at least two systems: the observed and the observing systems. The observed system is the subject system being studied (lets call the observed systems, S). The observing system is the observer, in this case us lets call the observing system, O).

For the observer, there is the modeling system comprised of ideas, knowledge and experience, etc. Recall that, ideally, the goal of the model is to know the exact state of the subject system at any point in time. When very little is known about the subject system, the observer's modeling system is at maximal diversity - there is no ability to determine the state of the subject system. This corresponds to the 1/n relationship of each mental model state being random. As more is learned about the subject system, the observer's model begins to (hopefully) determine the states of the subject system with greater accuracy.

The ability of the observer's modeling system to determine the observed system's states is a result of the observer's modeling system acquiring and representing the constraints of S. The importance of the constraints is that they can be used to determine the state of S. The constraints can take two basic forms. At the level of the architecture of the observer's model the constraints, or symmetry, take the form of the variables that define the system. The diversity of the observer's model is constrained by defining those factors that describe the behavior of the observed. Next, symmetry further impacts the observer's model by determining the degree to which the variables change. The observer's own model becomes more accurate as it becomes more symmetric to, and has greater symmetry with the observed.

When the observer's mental model can precisely determine the state of the subject system at any point in time, then the observer's model has reached its own maximal symmetry with respect to the subject system. The symmetry and diversity of S are independent of the symmetry and diversity of O. And so, when we speak of symmetry and diversity with respect to a subject system, keep in mind the parallel systems of observer and observed.

2.11 The Diversity Equations

In this next section we explore in some greater detail the relationship between the probability distribution of the state space and the quantity of base-two-machine-encoded information. First, let's set up another thought experiment. This time we will work with five similar abstracted systems. We will be looking at the variables, the possible states of each variable, the state space of each system, and lastly, the probability distributions of each state space. In Table #1, you will note, the systems are basically identical except for their state space frequency distributions. Keep in mind that these abstract representations could be of the same system at different times, or different systems at the same time. Also, the following models, which are statistical in form, could represent that stage of a system modeling study where the state of understanding on the part of the observer is limited to only to sets of systematic observations. From these observations, statistical distributions amongst the variables are then calculated. These distributions constitute the extent of the observer's knowledge about the system.

For these example systems, some of the variables are binary variables (remember? these are variables that exist in only one of two states, like on/off, up/down, 0/1, etc). The binary nature of these variables is not in any way required for the base-two-machine-encoding required for determining the quantity of information. These variables could have many more states, and we will present examples of such later on.

In **Table 2.1** we show the probabilities of each system *variable state*. Note that the probabilities for each variable state total to unity (=1). For instance, variable A in System 1 can exist in state [0,1] with respective probabilities of 0.5 + 0.5 = 1.0. Notice also that System 1 has equal probabilities for each variable state. This condition is characteristic of a random system. Next, notice that there is a trend from System 1 to System 5. For example, if you inspect the trend in variable probabilities for the state, 0, you will notice that state going from 0.5:0.5 (random) in System 1 to 0.99 certainty in System #5. Another way of expressing this is:

$$P_{A1}(0) = P_{A1}(1) = 0.5 \quad \text{and} \quad P_{A5}(0) = 0.99 \neq P_{A5}(1) = 0.01$$

Table 2.1: System Variable Probabilities

Variable	State	Sys #1	Sys #2	Sys #3	Sys #4	Sys #5
A	0	0.500	0.625	0.750	0.875	0.990
	1	0.500	0.375	0.250	0.125	0.010
B	0	0.500	0.625	0.750	0.875	0.990
	1	0.500	0.375	0.250	0.125	0.010
C	0	0.333	0.500	0.666	0.833	0.990
	1	0.333	0.250	0.167	0.083	0.005
	2	0.333	0.250	0.167	0.083	0.005

This trend is even clearer in **Table 2.2**, below. In this table we show the probability of each *system state* occurring. The state probability is simply the product of each variable state. For instance: take the event, or state, 000. The probability of this event in system #1 is:

$$P_1(000) = P_{A1}(0) * P_{B1}(0) * P_{C1}(0) = (0.50) * (0.50) * (0.33) = 0.083$$

Again, note the probability distribution for System #1. Note that for the random system, each state is equally probable. Also, note that the sum of each state probability for each system equals unity (or very close, due to rounding errors).

Table 2.2: System State Probability Distributions

State ABC	Sys #1	Sys #2	Sys #3	Sys #4	Sys #5
000	0.083	0.195	0.375	0.637	0.970
001	0.083	0.098	0.094	0.064	0.005
002	0.083	0.098	0.094	0.063	0.005
010	0.083	0.117	0.125	0.091	0.010
011	0.083	0.059	0.031	0.009	0.000
012	0.083	0.059	0.031	0.009	0.000
100	0.083	0.117	0.125	0.091	0.010
101	0.083	0.059	0.031	0.009	0.000
102	0.083	0.059	0.031	0.009	0.000
110	0.083	0.070	0.042	0.013	0.000
111	0.083	0.035	0.010	0.001	0.000
112	0.083	0.035	0.010	0.001	0.000
Sum:	0.999	0.999	0.999	0.998	1.000

Before we move on, careful attention must be placed on the fact that all of the forgoing discussion is predicated on the assumption that all of the variables are independent of each other. In other words, there are no cause-effect relationships between the variables. If there were any kind of dependence between the variables, then a more complicated process would be required to calculate the probability distributions.

2.12 Probability and Bits of Information in a Message

Once we have the probability distributions calculated for the state spaces we can now proceed to the next step. What we are basically interested in is to link probability to number of bits - which is base-two-encoded information.

Based on the discussion above, what you might guess is that we can take the probability of a particular state, or event, and apply the log2 function, something like: log2 of the state probability. Well, this is very close. One small problem is that, for example, take the probability of state 000 of system #1, which is 0.083. If you just take the log2 of 0.083 then you would have: log2(0.083) = -3.59 bits. OK, the notion of negative information does not make much sense for us given the context of this discussion. But, we do want to keep the same bit quantity of information. So, there is a mathematical way to convert the negative sign: take the log2 of the reciprocal of the state probability. From our example, this would mean 1/0.083. Now we have:

$$\log_2(1/0.083) = +3.59 \text{ bits.}$$

To summarize, what we now have is:

$$\log_2(1/Pe)$$

This is the number of bits associated with the probability of a given state, where Pe is the probability of a certain event, e, or state, occurring. **Table 2.3** summarizes all the of the bit values for each state of each system. But, what does this really mean?

Table 2.3: Information Content of Message Indicating System State, or the Potential Surprise (units are bits), $\log_2(1/Pe)$

State ABC	**Sys #1**	**Sys #2**	**Sys #3**	**Sys #4**	**Sys #5**
000	3.586	2.358	1.416	0.650	0.043
001	3.586	3.357	3.416	3.972	7.673
002	3.586	3.357	3.416	3.985	7.673
010	3.586	3.095	3.001	3.457	6.673
011	3.586	4.094	5.001	6.779	14.302
012	3.586	4.094	5.001	6.792	14.302
100	3.586	3.095	3.001	3.457	6.673
101	3.586	4.094	5.001	6.779	14.302
102	3.586	4.094	5.001	6.792	14.302
110	3.586	3.832	4.586	6.264	13.302
111	3.586	4.831	6.586	9.586	20.932
112	3.586	4.831	6.586	9.599	20.932
Sum:	43.032	45.132	52.012	68.112	141.109
Average:	3.586	3.761	4.334	5.676	11.759

One approach to understanding the above values is to realize that when we are associating probabilities of states occurring with bits of information we are referring the information content of a message transmitted from the sender to a receiver. This message has only one purpose: to indicate the state of system. Lets look at some of the data in Table #3 and consider the information value of messages indicating which state has occurred.

To begin, compare Systems 1 and 5. In System 1 each state is equally probable, so we expect the bits per state message to be equal, and indeed they are. But, System 5 is quite different. Specifically, note the message bit values for states 000 and 100. Notice that state 000, which is the most probable, has the smallest bit value than any other state for that system. Also, notice the converse, the state with a much lower probability of occurring (P(100) = 0.01) has a correspondingly much greater number of message bits associated with it. Basically, what this boils down to, is that we can say the same thing two ways: the greater the probability of an event occurring, the less information we associate with the knowledge of the event occurring. Conversely, the less likely an event is to occur, the greater the value of a message indicating that the event has occurred.

Another way of thinking about events that are less likely to occur, is that they are the events that would surprise us the most. That is, since they are highly likely to *not* occur, then when they do occur, they surprise us more than events that are more likely to occur. This surprise is associated with messages conveying more bits of information. Also, since we are basing our calculation on data collected from observations (and from these observations we derived a frequency distribution which in turn led to an expected or potential frequency distribution), the surprise we are talking about is thus expected to happen. So, we can give the name Potential Surprise, PS, to the expression $\log_2(1/Pe)$, or

$$PS(e) = \log_2 (1/Pe).$$

One important distinction needs to made at this point. When we say that the information content of a message is "worth" so many bits, we are NOT talking about the subjective worth of that information - just the base-two-machine encoded information that is related to state space probability distributions. The subjective value of a message is the receiver's perception of the message's importance and is obviously very difficult to measure. While the message's importance can be a very important real-life issue, the bit value of message information is simply limited to the probability distribution of the system state space. Recall that one of the driving factors behind choosing a particular encoding system for machines (i.e., computers) was that it has to be least cost. Cost is critical when resources for communication are limited. Thus, another implication of least cost is how and when data is communicated, or transmitted between sender and receiver.

Consider, the following:

> *You are a deep-sea explorer conducting deep-sea research with a state-of-the-art deep-sea submarine. You are exploring the far reaches of a yet-unexplored region of the ocean depths. You have a limited supply of battery power that is available for sending and receiving messages to and from your base ship on the ocean surface. On the second day of a five day mission, you receive the highly disturbing news that your mother has suddenly become gravely ill, and is not expected to survive. You naturally decide to abort the mission in order for you to go and visit her. But, your batteries are very low. Your dilemma is: given what you now know, what would be the most critical information for ensuring that you do what you can for your mom and, at the same time, not sacrifice the mission, if at all possible?*
>
> *Since you expect your mom to not make it, receiving a message that she has passed away would not cause you to change your mind for aborting the mission - as you must go home anyway for the funeral and memorial services. Also, such a message would consume valuable and limited battery power. Given the constraint of limited communication capacity, the most valuable information that you (and your mom) could receive would actually be that she has had a miraculous recovery, is not in danger of dying and can wait for you to finish your mission. After your mission is completed, then you can go and visit her.*

The above example illustrates that the amount of information in a message is related to the probability of the corresponding event because the greater the expectation of the event occurring the less we gain from the message. The subjective perceptions of the message information are real and important - just different from the bit value of information content. At the limit, a message about an event that has a 100% probability of occurring has no information because you already know that it will happen - so, you do not gain anything from this kind of message. Conversely, a message indicating that a highly improbable event has occurred will have great information content because it was not expected to occur, and thus had lots of surprise - so, you could gain a great deal from this kind of message.

2.13 From Surprise to Diversity

Referring to Table #3, above, we might get the impression that the messages from System #5 have more information than System #1. But, in fact, although this might be true in certain cases, on the average, Systems #1 - #4 will have more information content than System #5. This is because we need to account for the probability of messages of certain sizes being sent. Recall that from the Potential Surprise equation, those messages with greatest information content are the least likely to be sent.

What we seek is a quantity (whose units are still bits) that represents a link between the total combined surprise of each state space and the diversity for the entire system. Since we know that the random system has both maximal diversity and maximal surprise we want to combine both the probability of the state occurring and the information content of the message about the state. This can be accomplished with the following expression:

$$\text{System Diversity} = \text{SD} = \sum_{i=1}^{n} [\, P_i * \log_2 (1/P_i)],$$

where P_i is the probability of state, i, in a system with a state space of, n, states. Again, recall that the units are in bits.

Referring to Table 4, we find the system diversity values for each system at the bottom of each column. There are some interesting trends. First, notice that the random system #1, has the greatest number of bits of system diversity. For System #5, the diversity has diminished significantly. The trend is obviously towards zero as the occurrence of only one state reaches a probability of one (in the case of System #5, this would be state, $P_5(000) = 0.97$). Thus, what we find is that the random state system represents the upper bound of any system's possible diversity. Each system and the variables that comprise it will have its own unique upper bound. As the system acquires more symmetry, the diversity will shrink towards zero. Note that what we expect from all closed systems is that they tend towards greater symmetry. And so, we would also expect their diversity to tend towards fewer and fewer bits. Refer **Table 2.4**.

Table 2.4: System Diversity: SD = Σ [Pi* $\log_2$ (1/Pi)]

State ABC	Sys #1	Sys #2	Sys #3	Sys #4	Sys #5
000	0.299	0.460	0.531	0.414	0.042
001	0.299	0.328	0.320	0.253	0.038
002	0.299	0.328	0.320	0.252	0.038
010	0.299	0.362	0.375	0.315	0.065
011	0.299	0.240	0.156	0.062	0.001
012	0.299	0.240	0.156	0.061	0.001
100	0.299	0.362	0.375	0.315	0.065
101	0.299	0.240	0.156	0.062	0.001
102	0.299	0.240	0.156	0.061	0.001
110	0.299	0.269	0.191	0.081	0.001
111	0.299	0.170	0.069	0.012	0.000
112	0.299	0.170	0.069	0.012	0.000
Sum:	3.583	3.407	2.873	1.901	0.252

2.14 From Diversity to Symmetry

As mentioned above, we know that we can measure the total amount of symmetry in a system by using the Fundamental Equation. To do this, we would use the form:

Total Symmetry = Diversity of Combinatorial Space - Diversity of State Space

So, referring again to our example of Systems #1 - #5, for the total diversity of the combinatorial space for each system we have System #1 because it represents the condition of maximal diversity: the random system where each state has an equal probability of occurring. Then, to determine the diversity of the state space we use the system diversity equation summarized in Table #4.

For system #1 we calculate the total symmetry as follows:

Total Symmetry of System #1 = [Diversity of Combinatorial Space] - [Diversity of State Space] = 3.583 - 3.583 =
= 0 bits.

OK, so what did we just do? Recall that System #1 represents the random system with maximal diversity, which means minimal, or no, symmetry. The result of 0 bits, makes perfect sense then.

For System #2 lets do the same thing, again using the data from Table #4:

Total Symmetry of System #2 = [Diversity of Combinatorial Space] - [Diversity of State Space] = 3.583 - 3.407 =
= 0.176 bits.

The symmetry calculations for the remaining systems can be summarized:

Total Symmetry of System #3 = 3.583 - 2.873 = 0.710 bits
Total Symmetry of System #4 = 3.583 - 1.901 = 1.682 bits
Total Symmetry of System #5 = 3.583 - 0.252 = 3.331 bits

To further illustrate the impact of shifting variable probabilities and the emergence of symmetry, the following Table #6 summarizes the probabilities associated with state 000 for each of the systems. The significance of state 000 is that it was arbitrarily chosen to become progressively more and more biased (or favored) to occur, within each of the Systems #2 - #5. Tracing the values in column indicates this characteristic: STATE Probability. There the probability of state 000 occurring begins with the random case in System #1 and becomes an almost certainty in System #5 with a P(000) = 0.97 or 97% chance of occurring. What is not explicit in **Table 2.5** is that for state 000 to become more biased, other states had to become less probable - because each total system probability has to sum up to unity (=1). Also, note how the system symmetry increases as the certainty of state 000 occurring becomes more and more probable.

Table 2.5: Summary of State 000 and Overall System Diversity/Symmetry.

System	STATE Probability	SYSTEM Diversity (bits)	SYSTEM Symmetry (bits)
#1	0.083	3.583	0.000
#2	0.195	3.407	0.176
#3	0.375	2.873	0.710
#4	0.637	1.901	1.682
#5	0.970	0.252	3.331

2.15 The Distribution of Symmetry: Mutual Information

We can now proceed another step towards developing our understanding of the above systems in terms of their symmetry. What we are most interested in at this stage is quantifying the degree of relationship each variable has with each of the other variables. Also, we want to know if groups of variables "move together", and if they do, can we estimate approximately how much they move together, or interact. The magnitude of interaction between variables is referred to as *mutual information* (MI). Another way of summarizing these issues is: once we know the total symmetry of the system, how can we determine its distribution amongst and between the system variables?

First, we are interested in determining which variables, or groups of variables interact. There are many possibilities resulting from different variable groupings. When two or more variables interact, we represent the interaction as a pairing of variables. We can view the individual interacting variables and the interacting variable groups as subsystems of the observed system. Thus, a subsystem comprised of a group of variables can have its own state, which results from the combination of individual variable states. These potential interactions can also be represented in the following graph, **Figure 2.2**. In this graph each corner, or node, is a variable, and each line connecting the nodes, or arc, represents a potential interaction.

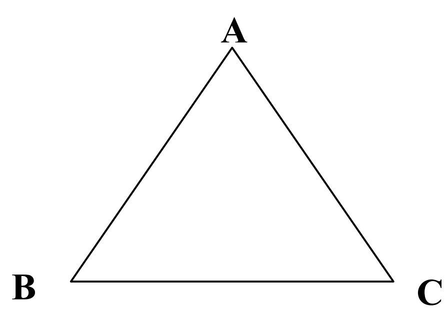


Figure 2.2 Graph of Three Variable System, ABC.

For this graph, each arc refers to the interaction of a pair of variables. The arc also represents the MI shared between the variables. Note that the arcs are undirected - that is they do not specify a direction. Most important however, is that each variable has an associated set of possible states. Let's represent just one possible state for each of the three variables from Figure 2 as follows: ***A*, *a*; *B*, *b*; *C*, *c***, for each variable and state, respectively.

To discuss MI, we need to introduce some new notation. What we need is to be able to say: "If you know the state of ***A***, then what are the chances of a certain state of ***B*** happening?" This question can be summarized succinctly as MI(***b*:*a***).

Equivalently, we could say the "If I know the state of ***A***, how much of my uncertainty about the state of ***B*** is eliminated (or reduced)". To get a better feel for this concept, consider the two extreme cases: random states that are completely independent of each other and two events that are linked via a cause-effect relationship.

First, think of throwing two "perfectly fair" die in a "perfectly fair manner". Based on everything we know about dice, the outcome of the *n-th* throw is not affected by the previous, or *n – 1,* throw. Each throw is not at all determined by previous throws. In this, if the first throw is ***a*** and the next throw is ***b***, then MI(***b:a***) = 0.

Now, think of an egg rolling off of a counter top and hitting the floor. Let ***a*** be the event and state of ***A*** for "egg rolling off of counter" and ***b*** be the event and state of ***B*** for "egg broken on floor". Ok, given ***a***, what do you know about ***b***? Given that you are certain that ***a*** has occurred, what do you know about ***b*** occurring? In this case we have:

MI(***b:a***) = 1.

For Figure #2, consider the arc between ***A*** and ***B***. We can correctly say that MI(***a:b***) or MI(***b:a***). In fact, it can shown that MI(***a:b***) = MI(***b:a***). The same applies to ***B*** and ***C***, and ***A*** and ***C***. This would also apply if the system had 30, 300 or 3000 variables all interacting to varying degrees.

2.16 Mutual Information and the Distribution of Symmetry

When the subsystems of a larger system are interacting with each other, they are "communicating" a certain quantity of information. In so communicating, they are also "sharing" the information. This sharing of information effectively results in the shared information becoming mutually "owned" by the interacting variables - becoming the link between the variables as mutual information. The phenomenon of mutual information is another aspect of symmetry. With MI, knowing the probability of one subsystem's state increases the observer's ability to accurately predict the state of other subsystems.

There are a few corollaries to this notion of MI between subsystems. First, if a subsystem does not share much information with any other subsystems belonging to the observed system, then it would not be a vital part of the model and can be excluded. This can be a good thing because it reduces the number of variables the observer has to contend with. Conversely, if a subsystem nearly or fully determines the states of other subsystems, then those other subsystems can safely be removed from the model.

Next, is a recapitulation of the previous discussions on diversity. We know that mutual subsystem information is symmetry. If we can calculate the quantity of information being shared, then we have a direct measure of the amount of symmetry within the observed system. Finally, we know that certain states can have a greater probability of occurring than other states. With this we can infer that certain variables have frequency distributions that are biased towards the favored state. In turn, we may want to know how the system symmetry is distributed amongst these variables and/or subsystems.

As hinted at above, the intuition for this concept is: given I know the state of B, what is the state of A? Another way to ask this question is: how much does the state of B determine the state of A? This question can be summarized in the following expression:

MI(A:B) = PS(A) - PS(A|B).

In English, this expression says:

"the mutual information between subsystem A and subsystem B
equals
the potential surprise of the state of A
minus
the potential surprise of the state of A given I know the state of B".

This expression breaks down into several components. To begin, we already know how to calculate the PS(A) and Table #4 summarizes the results these calculations. Next, there is the calculation for the PS(A|B). This calculation involves a multi-step process. Here we go...

2.17 Delving in PS(A|B)

Recall that the basic notion of PS is stated in the expression

$$PS(e) = Pe*log_2(1/Pe).$$

Its the term Pe that we need to focus on right now. One of the key characteristics of the term Pe is the following:

$$\Sigma\, Pe = 1.$$

This expression states that the sum of the probabilities for all of the possible events, e, of the system must be 1. So, if we know that B exists in either one of only two states: B=0 or B=1, and that the chance of P(0) = 0.5, then we automatically know that since

$$P(B=0) + P(B=1) = 1, \text{ then}$$

$$1 - P(B=0) = P(B=1)$$

$$1 - 0.5 = P(B=1)$$

$$0.5 = P(B=1)\,.$$

As we continue to use the expression for calculating PS, we will need to pay special attention to how we determine Pe.

To continue with the steps for calculating PS(A|B), what we are interested in is calculating the degree to which B determines A. In order to do this we must first determine what the possible state combinations are for A when B is in a given state. Then, we determine what the associated probabilities are for each of these states. Each of the probabilities for these states is how we determine Pe.

We want to know the states of A given we know the states of B. Because each of these variables has only two states, there are four possible conditions:

A = 0 and B = 0
A = 1 and B = 0
A = 0 and B = 1
A = 1 and B = 1

In each of these cases, there is an associated probability. So, lets summarize what we have so far as follows, where Pe is the probability of event, or state, e, and e: [1, 2, 3, 4].

P(A = 0, B = 0) = P(00) = P1
P(A = 1, B = 0) = P(10) = P2
P(A = 0, B = 1) = P(01) = P3
P(A = 1, B = 1) = P(11) = P4.

These data can also be represented in the following schematic, **Figure 2.3**.

```
                         |---P(A=0|B=0) = P1
                         |
                         |
          |----P(B=0)----|                            P1 + P2 = 1
          |              |
          |              |
          |              |---P(A=1|B=0) = P2
          |
          |
PS(A|B)---|
          |
          |
          |              |---P(A=0|B=1) = P3
          |              |
          |              |
          |----P(B=1)----|                            P3 + P4 = 1
                         |
                         |
                         |---P(A=1|B=1) = P4
```

Figure 2.3 Schematic of PS(A|B)

Each of the states of B (i.e., B=0 and B=1) represent a main branch in the above schematic. From each main branch, there is a sub-branch that represents the states of A with a given state of B. So, we are interested in the probability of the system being in a state represented by one of the sub-branches P1...P4. Before we attempt to determine the probabilities associated with each of the sub-branches it would be a good idea to first determine what the overall probabilities are for each main branch.

We know that

$$P(B=0) + P(B=1) = 1.$$

From this we can see that

$$P(A=0|B=0) + P(A=1|B=0) = P1 + P2 = 1.$$

The same is true for P3 + P4 = 1. To determine the value of any one Pe (that is, a specific sub-branch), we will need to determine the probability of the sub-branch itself. So, the question arises: how is the probability of a specific sub-branch determined given that all we know is P(A) and P(B)?

For the B=0 main branch we can use the following expression of the A=0 sub-branch:

$$P(A=0|B=0) = P1 / (P1 + P2).$$

This expression works because we know that

$$P(A=0|B=0) + P(A=1|B=0) = 1, \text{ and because} \quad P1 / (P1+P2) \quad + \quad P2 / (P1+P2) \; = 1.$$

The sub-branches that branch off of a main branch (i.e., B=0) must have probabilities that equal unity. So, we use the ratio of the individual event probability and the sum of all event probabilities for each event Pe.

Now that we have the basic idea for determining the Pe for each state of A given that we know the state of B, the PS(A|B) looks like this for each of the four states:

$$PS(A|B) \quad = \quad Pe \quad * \quad \log_2(1/Pe)$$

$$PS(A=0|B=0) = [\ P1 / (P1+P2)\ ] * \log_2(\ P1+P2 / P1)\ )$$
$$PS(A=1|B=0) = [\ P2 / (P1+P2)\ ] * \log_2(\ P1+P2 / P2)\ )$$
$$PS(A=0|B=1) = [\ P3 / (P3+P4)\ ] * \log_2(\ P3+P4 / P3)\ )$$
$$PS(A=1|B=1) = [\ P4 / (P3+P4)\ ] * \log_2(\ P3+P4 / P4)\ )$$

The complete expression for PS(A|B) takes the following form:

$$PS(A|B) = P(B=0)*[\ P1 / (P1+P2)\ ] * \log_2(\ P1+P2 / P1) \ + P2 / (P1+P2)\ ] * \log_2(\ P1+P2 / P2)\ ] +$$
$$P(B=1)*[\ P3 / (P3+P4)\ ] * \log_2 (\ P3+P4 / P3) + P4 / (P3+P4)\ ] * \log_2 (\ P3+P4 / P4)\ ]\ .$$

(...recall that our goal so far is to calculate the MI(A:B), and that PS(A|B) is just one part of equation...)

With the forms of the PS expression worked out, now we need to figure exactly what these probabilities are for each Pe. To accomplish this we need to refer back to Table 2.2. For the purposes of illustrating this point we will only be using data for Systems #1, #3, and #5

Table 2.2: System State Probability Distributions

ABC	State	Sys #1	Sys #3	Sys #5
000		0.083	0.375	0.970
001	P1	0.083	0.094	0.005
002		0.083	0.094	0.005
010		0.083	0.125	0.010
011	P3	0.083	0.031	0.000
012		0.083	0.031	0.000
100		0.083	0.125	0.010
101	P2	0.083	0.031	0.000
102		0.083	0.031	0.000
110		0.083	0.042	0.000
111	P4	0.083	0.010	0.000
112		0.083	0.010	0.000
Sum:		0.999	0.999	1.000

The next step is to notice that for P1, P2, P3, and P4 there are three separate states. This is because of the presence of variable C: [0, 1, 2].To calculate the probability of state P1...P4 for each system we need to sum each of the three states. For example, the value of P1 for System #1 is 0.083 + 0.083 + 0.083 = 0.249 bits. **Table 2.6** summarizes this data for all of the other cases.

Table 2.6: Summary of Probability Values for States P1...P4.

State Probability	System #1	#3	#5
P1	0.249	0.563	0.980
P2	0.249	0.187	0.010
P3	0.249	0.187	0.010
P4	0.249	0.062	0.000
B = 0	0.50	0.75	0.99
B = 1	0.50	0.25	0.01

With the data from Tables #2 and #7 we can finally fill in the probability values we need for the PS(A:B) expression. For System #1 the PS equation looks like this:

PS(A|B)=0.5*[0.249/0.249+0.249)*(log2(0.249+0.249)/0.249)
+(0.249/0.249+0.249)*(log2(0.249+0.249)/0.249)]
+0.5*[(0.249/0.249+0.249)*(log2(0.249+0.249)/0.249)
+ (0.249/0.249+0.249)*(log2(0.249+0.249)/0.249) = 1

PS(A|B) = 0.5*[(0.5 * log2(2)) + (0.5 * log2(2))] + 0.5*[(0.5 * log2(2)) + (0.5 * log2(2))] = 1

PS(A|B) = 0.5*[(0.5 * 1) + (0.5 * 1)] + 0.5*[(0.5 * 1) + (0.5 * 1)] = 1

PS(A|B) = 0.5*[(0.5) + (0.5)] + 0.5*[(0.5) + (0.5)] = 1

PS(A|B) = 0.5*[1] + 0.5*[1] = 1.0 bit

For determining the value of PS(A) we have:

PS(A) = sum [0.5*log2(1/0.5) + 0.5*log2(1/0.5)] = sum [0.5*(1) + 0.5*(1)] = sum [0.5 + 0.5] = 1 bit

Finally, we can now work on MI(A:B):

MI(A:B) = PS(A) - PS(A:B) = 1-1= 0 bits.

Hmmmm, what does this mean? Well, recall that for System #1 we already knew that because it is a random system there will be no detectable symmetry and therefore no MI. So, the MI(A:B) = 0 bits is what we expect to happen.

Exercise:

Calculate the MI(A:B) for Systems #3 and #5.

2.18 Information and Energy

The properties of information are in certain ways similar to energy. In particular, the laws of thermodynamics applying to energy have their analogs that apply to information. These Laws all refer to closed systems, that is, systems that share only energy or information with their environments (and not matter).

2.19 The FIRST LAW - The Conservation of Energy and Information

This law states that the total amount of energy in closed system remains constant - what does change is the *form* of the energy. Similarly, the quantity of information in a closed system remains constant - however, it can change form. Recall that when we are speaking of information in this context, we are referring to number of bits required to encode the state space - not the subjective meaning of the information to the observer. For example, if the observed system happens to be a hard drive for a computer and certain vital data is accidentally deleted - the number of bits of data on the hard drive remains the same - unfortunately, they are all 0's.

2.20 The SECOND LAW - "Useful" Energy Always Decreases

While the energy in a system remains constant, the first law indicates that the form changes. Now add the characteristic of useful-ness to the energy we are concerned with. Diversity is that characteristic of energy and information that is useful and gives us something to “harness”. As we harness energy to do work, i.e., think, eat, travel, cook food, ad infinitum, we "use it up". Think about what this means. When we use energy to do work, two things happen. First, we get some work done. Second, we convert chemical, or mechanical, or electrical or gravitational energy into something else when we are done using it. The energy that we used to get the work done has been converted into another form of energy - even though we never intended the transformation to occur. This transformation results in the energy being converted into a less useful form of energy - usually in the form heat energy.

The second law states that each time work-energy is used, it is converted into a less useful form of work-energy. Another way to say this is that the work-energy becomes more and more use-less as it does work. Another word for useless-ness is *entropy*. Now we can say that the work-energy of a system can only increase in entropy.

Symmetry and entropy are analogs of each other as are information and energy. The symmetry of a system can never decrease - it can only stay the same or increase. In other words, the number of bits required to encode the state space can never increase - they can only stay the same or decrease. Again, referring to hard drive example, without the some kind of input from an external system - i.e., the user - the storage device will never obtain a greater capacity for storing data - just as the work-energy of a system will never obtain a greater capacity to do work without the input from an external system.

2.21 The THIRD LAW

This law deals with the extreme of no energy in a system. This phenomenon corresponds to absolute zero. At absolute zero all atomic movement stops. Nothing changes position. This is the analog of symmetry - perfect symmetry where everything is the same. A system at "absolute zero" can be thought of as being perfectly predictable - no surprises. The current state and all future states are known simply because the system is "frozen" into only just one state. This state repeats over and over again at each moment in time.

2.22 D-S-N Graph

The Diversity-Symmetry-N Graph provides a means of describing all systems. With this graph (refer **Figure 2.4**) we have the horizontal axis which represents N, the total number of possible system states, that is, the combinatorial space. At the origin, we begin with a single state and from there we expand into an infinite number of states. The graph then plots the $\log_2(N)$.

The left-hand vertical axis represents the $\log_2(N)$ bits of diversity ranging from 0 to infinity. On the right-hand axis, going in the opposite direction, we can also have symmetry with a lower bound - 0 - at the "top" of axis and increasing to an upper bound. The upper bound of symmetry, Smax, will depend on the particular system being represented on the DSN graph.

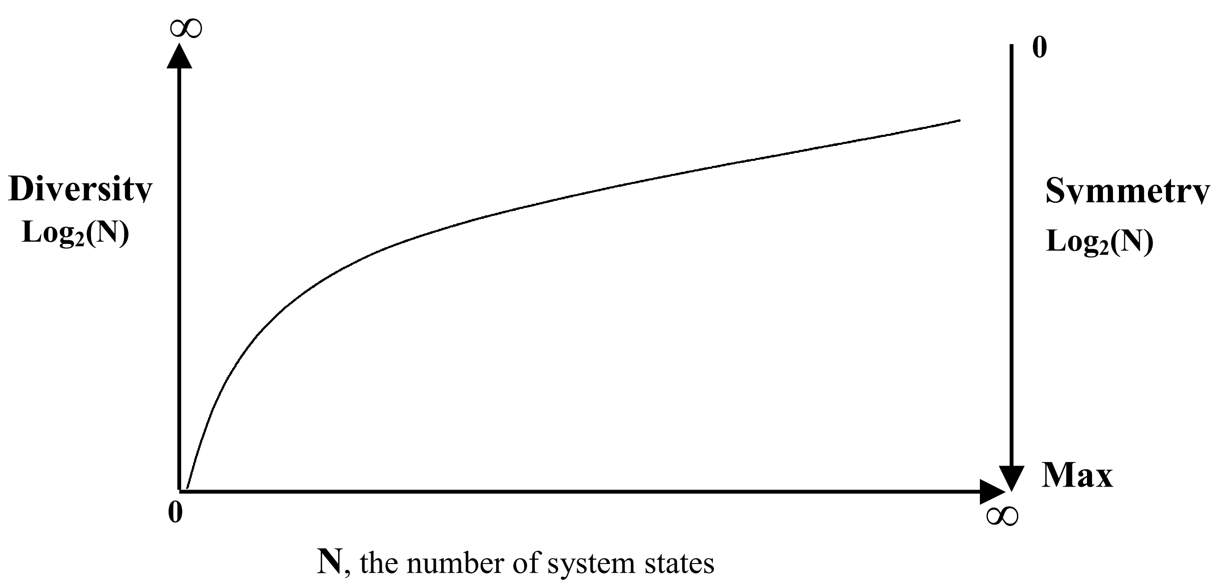


Figure 2.4 The DSN Graph

All systems can be represented somewhere on the $\log_2(N)$ curve. With the DSN graph, we can compare many systems to each other and to themselves as they change with time. As systems evolve through time, their path of decreasing diversity (increasing symmetry) will be traced along this curve.

2.23 A Roulette Wheel, a Die and a Coin

If we compare a roulette wheel, a die and a coin together on the DSN graph, we can gain some insight into different systems. First, the roulette wheel where N = 36 and D = $\log_2(36)$ = 5.17 bits. Next, the die has an N = 6 and D = $\log_2(6)$ = 2.58 bits. Finally, the coin has an N = 2 and D = $\log_2(2)$ = 1 bit (refer **Figure 2.5**).

Now, lets imagine the you are gambling away your tuition money by playing the roulette wheel. You want to play the number 36. So, you have a 1 in 36 chance of winning. Now lets say you could play a group of numbers, such as {1, 6, 12, 18, 24, 36}. In this case, your chances of winning are 1 in 6. From the standpoint of diversity, you could be achieving the same thing by rolling a die {1, 2, 3, 4, 5, 6}. Alternatively, you could only play the red or black spaces on the wheel. The chances of winning are now 1 in 2. Or, just flip a coin for the same chance of winning. These examples demonstrate how a system with N = 36 can be restricted to behave identically to systems with N = 6 and N = 2. When observing a system, it is often useful to keep in mind that the state space may be a very small subset of the potential magnitude in the system's combinatorial space.

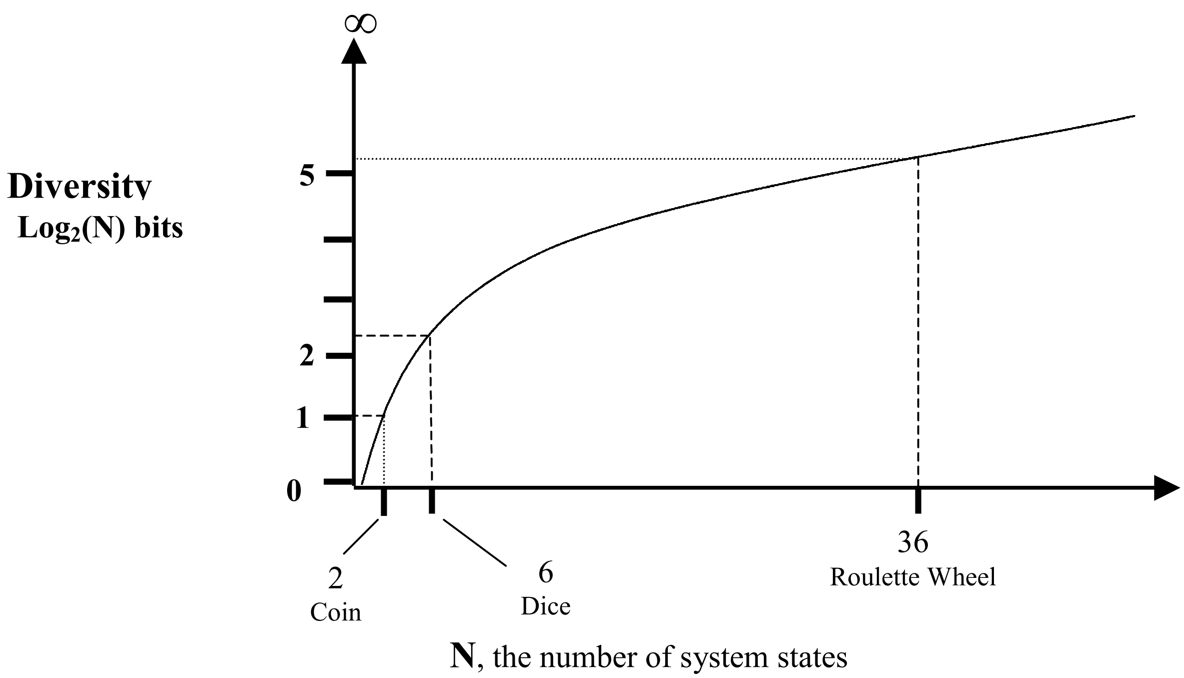


Figure 2.5: The DSN Graph for Roulette, Die and Coin Systems

2.24 Equilibrium

As mentioned above, the DSN graph provides a convenient way of tracing the evolution of a particular system through time. The system can begin with N equaling its combinatorial space for possible states to exist in. As the system passes through time, the number of reachable states begins to diminish as diversity is lost due to the increasing number of unreachable states. Eventually the system will evolve to a stable number of states. The key thing to look for is that the variables will stop changing values or that their values will be repeating a certain set of values. In this situation, the system's state space can be either a single reachable state (i.e., D = 0 bits = log2(1)) or a set of repeating states. At this point the maximum symmetry would be achieved (Smax) and would equal the lower bound of the system 's original diversity. In the case of a repeating set of equilibrium states, the system's state space could still retain a significant fraction of N. For this case then, the maximum S would be somewhere in-between D = 0 bits and Dmax = $\log_2(N)$.

When a system does eventually reach a stable number of states, the system can be described as being in *equilibrium*. What is essentially happening for a system in equilibrium is that the same state or sets of states are simply repeating themselves. If the system's equilibrium state space is a single repeating state, the equilibrium is referred to as *static equilibrium*. If the equilibrium state space has more than one state in it, then, this condition is referred to as *dynamic equilibrium*.

A good question at this point is: exactly how do symmetry and equilibrium come into existence? After all, it has been stated almost axiomatically that symmetry and equilibrium will occur, especially in closed systems. But, the question now is: how?

2.25 Transition Graphs and Transition Probabilities

We now introduce another type of system picture, or graph, for the purpose of illustrating the evolution of a system towards greater symmetry and equilibrium. To begin, let's imagine a system with a finite number of states. Let each state be represented by a dot, or node. As the system changes in time, it is changing, or transitioning, from one state to the next. Let the line, or arc, between each node represent the idea "changes into". In other words, if a system goes from state s1, to state s2, we can represent this transition with two nodes connected by an arc.

Now refer to **Figure 2.6**. Here we are representing a system with N=5 states: { s1, s2, s3, s4, s5 }. The system exists in two time periods, T_1 and T_2. At T_1, every state { s1...s5 } is capable of transitioning to every state { s1...s5 } at T_2. This omni-connected-ness is represented by each of the 25 arcs connecting the 5 states at T_1 to each of five states at t T_2 (25 = N^2). When we say that all states at T_1 can transition to T_2, we are saying that each transition is possible.

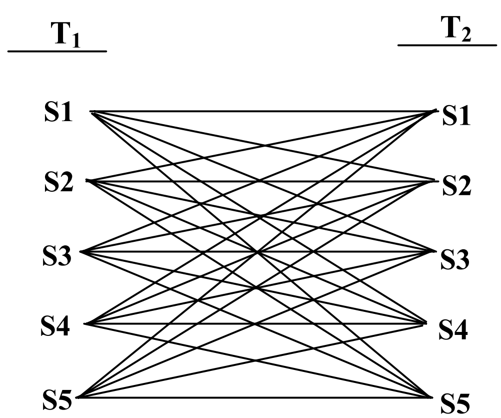


Figure 2.6: Complete Bipartite Graph

When talking about possibilities, it is often useful to think in terms of probabilities. We can express probabilities of states occurring as follows:

$$P(s1) = 0.5$$

This says that "the probability of state s1 occurring is 0.5, or 50%". If we want to say that state s1 transitions to state s2, we can express this idea as follows:

$$P(s1,s2) = 0.5$$

Here, the idea of transitioning from one state to another is indicated by the indices, i and j, for "before and after", which gives: si,j. The index, i, is for T_1, and j, is for T_2. The transition probability could be thought of as being the value of an arc. With this particular representation, the arcs are directed, meaning that they are "one way". This is due to the lack of any possibility of a state in T_2 going "backwards" in time to a state in T_1. If we wanted to, we could represent this type of transition, but for now lets just go with one- way-directed graphs. T_1, T_2

2.26 Transition Matrix

To summarize the transitions from T_1 to T_2, we can organize all of the probabilities into a table or matrix. Lets make each T_1 or i the rows and each T_2 or j a column. Refer to **Table 2.7**. In this table, start with row, i=1. What this table says is that when i=1 and j=1, there is a P(s1,s1) = 1/5. This means that there is a 1/5 chance that s1 will stay the same at T_2. For i=1 and j=2, P(s1,s2) = 1/5 which means that there is a 1/5 chance that s1 will go to s2.

Table 2.7, Transition Matrix for T_1 and T_2

				T_2		
		S1	**S2**	**S3**	**S4**	**S5**
	S1	0.2	0.2	0.2	0.2	0.2
	S2	0.2	0.2	0.2	0.2	0.2
T_1	**S3**	0.2	0.2	0.2	0.2	0.2
	S4	0.2	0.2	0.2	0.2	0.2
	S5	0.2	0.2	0.2	0.2	0.2

Note also that the sum of each row must equal unity (one). The sum of each column, on the other hand, can have values between {0...i=n), or equivalently between 0 and the number of rows. Referring back to Figure 6, we can assign to each arc the values in Table 8 for each transition si,j. For the matrix represented by Table 8, this is a system with maximal diversity since P(si,j) = 1/N = 1/5, and,

$$D = \Sigma \ [1/pi*\log2(1/pi)] = 2.32 \text{ bits.}$$

2.27 Transition Configurations

One-to-Many

Again, referring to Figure 6, let's focus on just s1 as it transitions from T_1 to T_2. This looks like **Figure 2.7**. The matrix for this transition is still represented in Table 8. In this case, only the top row applies. This type of transition is called the *one-to-many* configuration because the one state at T_1 can transition to multiple states in T_2. As long as S has more than one possible state at T_2, such that P(s1,j) > 0, then one-to-many can occur. This type of transition is referred to as stochastic when each of the states in T_2 has a different probability of occurring.

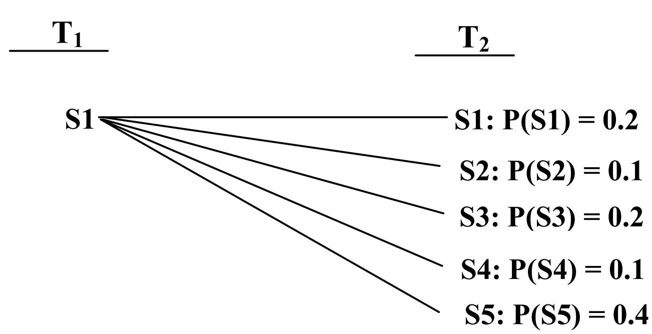


Figure 2.7: One-to-Many Graph

Many-to-One

If we flip things around a bit, we get the *many-to-one* transition configuration. Referring to **Figure 2.8**, multiple states at T_1 all transition to a single state at T_2. Again, Table8 would still show the probabilities for this transition. In this case, the probabilities in the column underneath S1 would correspond to those states in T_1 that could transition to T_2. As long as the system has more than one possible state at T_1 and only one state at T_2, such that P(si,1) > 0, then many-to-one can occur.

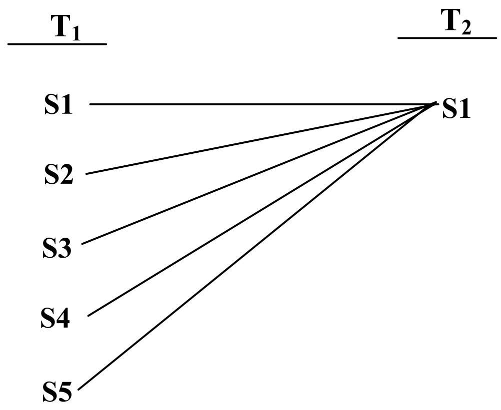


Figure 2.8: Many-to-One Graph

Many-to-Many

If we combine the above two configuration types we get the *many-to-many* transition configuration. Obviously, this case has the greatest diversity. Figure 6 represents the graph of a many-to-one transition. Table 8 represents all of the transitions for such a graph. Note that in this case, each P(sij) > 0.

2.28 The State Transition Series

All systems exist in time and have more than just one transition (i.e., from T_1 to T_2). We can represent a period of time as a series of time steps, T_1, T_2, T_3,..., T_n. For each time step we have the states, s1, s2, s3,...sm. For our example, lets keep the five states such that m=5. Referring to **Figure 2.9**, we have the time series $T_1 \ldots T_6$ going horizontally and the states s1..s5 arranged vertically. Linking each column of s1..s5 at t to the next column of s1..s5 at T_{j+1} are arcs. The transition associated with each time step has its own transition matrix, where i=t and j=t+1. Thus, the state transition series also has a corresponding succession of transition matrices with each matrix having its own probability distribution.

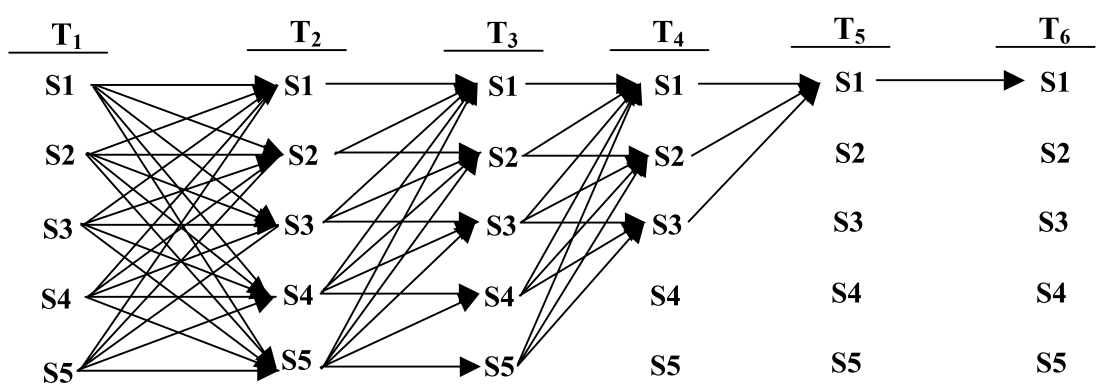


Figure 2.9, Transition Series

For example, for T_0 to T_1 refer again to **Table 2.7** above. This is the classical random, highly diverse system illustrating the many- to-many configuration where for t0- T_1, P(si,j) = 1/N = 1/5. Any initial state T_0 has an equal chance of transitioning into any state at T_1.

Now lets go from T_1 to T_2. In **Table 2.8**, we show the associated matrix for this transition. As you can see for si = {1,2,3,4,5} the probability of state s5 occurring at T_2 is no longer random, since P(si,1) = 0.6 > 1/N = 1/5. What this means is that no matter which state occurs at T_1, there is a strong probability (p(s5) = 0.6) that the next state at T_2 will be s1. In this type of situation, we have a many-to-one configuration developing.

Table 2.8, Transition Matrix for T_2 and T_3

T_2 \ T_3	S1	S2	S3	S4	S5
S1	1.0	0.0	0.0	0.0	0.0
S2	0.6	0.4	0.0	0.0	0.0
S3	0.6	0.2	0.2	0.0	0.0
S4	0.6	0.13	0.13	0.13	0.0
S5	0.6	0.1	0.1	0.1	0.1

Table 2.9, Transition Matrix for T_3 and T_4

T_3 \ T_4	S1	S2	S3	S4	S5
S1	1.0	0.0	0.0	0.0	0.0
S2	0.6	0.4	0.0	0.0	0.0
S3	0.6	0.2	0.2	0.0	0.0
S4	0.6	0.2	0.2	0.0	0.0
S5	0.6	0.2	0.2	0.0	0.0

Table 2.10, Transition Matrix for T_4 and T_5

T_4 \ T_5	S1	S2	S3	S4	S5
S1	1.0	0.0	0.0	0.0	0.0
S2	1.0	0.0	0.0	0.0	0.0
S3	1.0	0.0	0.0	0.0	0.0
S4	1.0	0.0	0.0	0.0	0.0
S5	1.0	0.0	0.0	0.0	0.0

Table 2.11, Transition Matrix for T_5 and T_6

T_5 \ T_6	S1	S2	S3	S4	S5
S1	1.0	0.0	0.0	0.0	0.0
S2	0.0	0.0	0.0	0.0	0.0
S3	0.0	0.0	0.0	0.0	0.0
S4	0.0	0.0	0.0	0.0	0.0
S5	0.0	0.0	0.0	0.0	0.0

2.29 Symmetry, Many-to-One and Determinism

In **Table 2.8** (for transition T_2- T_3), the probability distribution has changed again. In this case, every state at T_2 (i) has a p(si,1) = 1, or a 100% chance of transitioning to state 1. When a system exists in one state that has a probability of one for transitioning into another state, the system is characterized as being *deterministic.* The matrix for a deterministic system will be easily recognized because there will have only **1's** and **0's** in it.

Another important characteristic of **Table 2.8** is the resulting increase in symmetry that arises from the state space containing a state that has a greater probability of occurring. This is also true for **Tables 2.9 – 2.11**. Referring to **Figure 2.8**, it is evident that the state space of the system is getting "narrowed down" to a single state as the system moves from $T_2 – T_6$. At this point, we do not know what sort of constraints, or "forces" are responsible for the probability distribution associated with $T_2 – T_6$. But, what we do know is that the narrowing down of the state space is due to the types of transitions that are occurring.

Essentially what is going on with the system is a succession of transitions that continue to favor one state over any other state. This is especially true in the transition from T_4 to T_5 . This transition is a clear case of many-to-one. The previous few transitions ($T_2 – T_4$) are not exactly many-to-one. Instead, they have probability distributions that strongly favor state 1. Thus, it is the specific mechanism that simultaneously reduces the size of the state space and increases the system's symmetry, is a combination of transitions that have probability distributions that either strongly favor one state or, are deterministically many-to-one (which is characterized by a probability distribution that says the P(si, 1) = 1).

2.30 Symmetry, Equilibrium and One-to-One

With the given set of conditions described above, and the continued succession of time steps, the system will eventually evolve to a repeating pattern of one or more states. This is demonstrated in transitions $T_5 – T_6$. Here just one state repeats itself (s1). As mentioned earlier, this condition is referred to as *equilibrium.* The system would presumably continue to be in equilibrium until some external force provided some form of input to the system.

2.31 Many-to-One and Functions

The many-to-one transition configuration is an important one. We find that the definition of a mathematical function depends on the many-to-one pattern. For functions, the transition, or mapping, of the inputs (the domain) to the outputs (the range) of the function. The function describes and determines exactly how the domain maps onto the range explicitly with a formula (or mathematical "recipe") and is thus a deterministic system.

f(x) = y

Inputs	Outputs
x1	y1
x2	y1
x3	y1
x4	y1
x5	y1
x6	y3
x7	y3
x8	y3
x9	y3

2.32 Thermodynamics, Again…

We have, so far, discussed what kinds of transition patterns impact the dynamics of diversity and symmetry for any system. What we haven't discussed until now is: what determines or causes particular transitions for systems?

As you might guess, systems involving matter, energy, space and time will be governed by what we can call The Laws of Nature. One very important subset of the Laws of Nature is called the Laws of Thermodynamics. These laws describe the exact transition patterns for systems that in any way involve natural elements (matter, energy, etc).

The concepts of symmetry and diversity are very useful for describing the types of transitions characteristic of the thermodynamic properties of natural phenomena. In particular, these properties involve a few key aspects of natural phenomena.

The first key aspect is heat. Heat can be thought of as the "energy content of matter". Matter itself IS energy, just in a condensed form. At the material level of elemental matter, what makes each element unique is its "vibrational pattern". In addition to the energy contained in the vibrational pattern, there is the added energy absorbed by matter that originated in the environment surrounding the matter. This energy often takes the form of heat, or kinetic energy. The addition of heat causes matter to "vibrate" more.

From the standpoint of human perception, heat is experienced by our senses as the measurable phenomena called: temperature. Increasing the heat energy of matter is analogous to turning up the volume of a stereo. While the underlying pattern remains the same (ie, the music being played), the vibration is just "louder" because it contains more energy. Of course, there is a limit, where the addition of enough heat will cause the vibrational pattern of matter to be overwhelmed and be reduced to the state of plasma (a mish-mash of subatomic particles). The laws of thermodynamics deal with how heat flows within and between systems.

In the context of diversity and symmetry, heat and all other forms of energy can be described as sources of diversity for all systems. The overall energy content of a system, in the broadest terms, can be characterized as the rate of state change for that system. More precisely, energy can be expressed as the number of state changes per unit time. This description of diversity would apply specifically to kinetic energy, or the energy of motion. Potential energy can be characterized as possible, future changes for a system.

Potential energy (PE) is the stored energy in a system. Think of a spring that is compressed, or a battery that is all charged up, or a heavy object lifted up with a crane. In each of these examples there is some form of PE. For the spring, there is the mechanical energy of the spring which will "expand" along its axis of compression when released. The battery will power the portable CD player by "releasing" the electrical energy that was stored chemically. The heavy object will fall to the ground when released due the force of gravity "pulling" it to the earth.

Another interesting application of thermodynamics, is the description of the "information content" of a system. This corresponds to what we have discussed already as the bits of diversity for a system. When the state space occupies the entire combinatorial space of the system, the maximal information (quantified as bits of diversity) is required to encode all of the system states. Thus, the system "contains" maximum information. As the system evolves and the state space diverges from the combinatorial space, symmetry increases.

2.33 Thermodynamics and the Fundamental Equation (FE)

Recall that the FE describes the relationship between the combinatorial space and the state space of all systems. The FE can be summarized as follows. For all systems:

the diversity of the combinatorial space
must equal
the diversity of the state space
plus
the symmetry of the state space.

In more formal terms, we have

DTcs = Dtss + Sss

where,
DTcs = the total diversity of the combinatorial space,
Dtss = the total diversity of the state space
Sss = Symmetry of the state space

The terms of mutual information (MI), Sss is the MI shared between all of the system variables (V) contained within the system:

Sss = Σ[MI (Vx : Vy)], for all x,y and x <> y.

When we look more closely at the Dtss term we have:

Dtss = Dss * (Dk + Dp) where,
Dss = the diversity of state space = Σ [1/p * $\log_2$ (1/p)]
Dk = the diversity of kinetic energy
Dp = the diversity of potential energy

. The units for the above Dtss expression are:

Dtss: bits/sec
Dss : bits/state change
De : state changes/sec
Sss : bits/sec

bits/sec = [(bits/state change) * (state change/sec] + bits/sec

Using these expanded terms, and substituting for (Dk + Dp) the term De, we now have:

DTcs = Dss * (Dk + Dp) + Sss = (Dss * De) + Sss

Another more useful form of the above expression is:

Sss = DTcs - [Dss * (Dk + Dp)] = DTcs - (Dss * De)

2.34 The DSE Graph and the Fundamental Equation

Previously, we had examined the DSN graph to learn more about a system by the relationship between its diversity and its combinatorial space (N). From the FE, we now must include De = (Dk + Dp). These terms can be combined to express the energy, E, of the system - the Diversity- Symmetry-Energy graph. Refer **Figure 2.12**.

This graph displays a 2-D graph space that is given by the FE. Note that some other set of transition functions must describe how the system is to evolve, gain symmetry and lose diversity. All the FE does is represent the impact of these transition functions in terms of changes in Diversity-Symmetry and E.

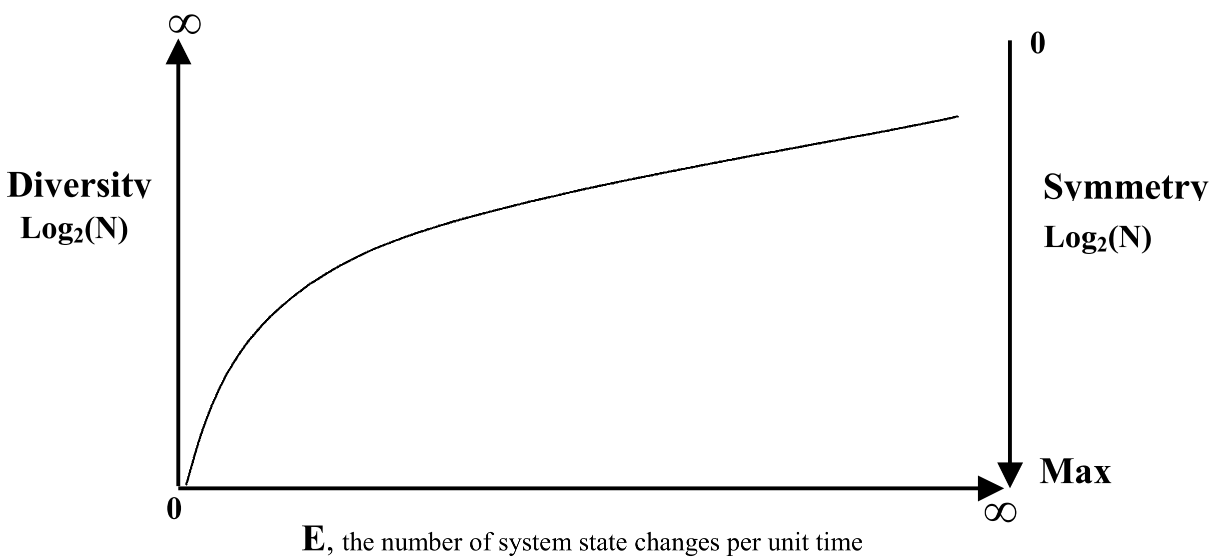


Figure 2.12, DSE Graph

2.35 The Zero-th Law

Returning to the Laws of thermodynamics, we begin with the less known Zero-th Law. This law deals with thermal energy being transferred between three objects that are both connected in such a way that thermal energy can be exchanged between the objects. In the context of diversity, thermal energy is represented by :

De: state changes per unit time.

In the literature of classical thermodynamics, the thermal energy content of a system is of key interest. Temperature is the measure of the macroscopic state of a system's thermal energy "content". When temperature differences exist between systems, and there are no barriers preventing the flow of thermal energy between the systems, then there exists a possibility of diversity (in the form of thermal energy) being exchanged between the systems. This flow of diversity is significant because it can be "tapped" into to do useful tasks by another system (i.e., work by Observed for Observer). Later we discuss the direction, rate and nature of the flow.

To get an intuitive sense of this Law, imagine three objects arranged sort of like three books on a shelf. The middle book has a special property. It acts as an intermediary between the two "outer" books. If we further imagine that the outer books have different temperatures, then the middle book can act to allow heat to flow through itself from the hotter to the cooler book until thermal equilibrium is reached (i.e., temperatures are equal and have stopped changing). The hotter system could be called "source" and the cooler book referred to as "sink".

The middle object has several additional properties. Firstly, after the two outer objects (systems) have reached equilibrium, the middle system returns to its original state. All that has happened is heat - or diversity - has flowed through it. Next, if the middle system is designed to convert heat flow into something useful (work), then the maximal amount of work accomplished will depend on the magnitude of the difference in diversity between the source and sink systems.

Diversity differentials do not solely depend on temperature differentials. Imagine two lakes. One of the lakes is located in an area at an altitude 1000 feet higher than the other “lower” lake. Between the two lakes is a large pipe that has a water powered electrical turbine generator arranged such that the flow of water will turn the turbines and make electricity. The differential in altitude corresponds to differences in gravitational force between the two lakes (systems). In this case, the lake at the higher altitude is the source and the lake at the relatively lower altitude is the sink. As we all know, the flow of water through the pipe will "naturally" flow from the upper to the lower lakes (source to sink, respectively). The electrical generator is the special intermediary system that converts the diversity differential into something-useful (i.e., electricity). The limit of diversity conversion will be achieved when equilibrium has been established between source and sink, or all of the water from the two lakes is at the lowest altitude (relative to the Earth's gravitational field).

It is this flow of diversity (especially thermal energy) combined with the conversion of this diversity differential between source and sinks by special intermediary systems that is the essence of the Zero-th Law. Also, integral to the Zero-th Law is that feature of natural systems where the amount of maximal useful diversity achieved by the intermediary system is limited by the diversity differential between source and sink before equilibrium is achieved. Thus, in the case of the two-lake system described above, the maximal amount of work possible is limited by the altitude difference between the two lakes. This is the maximal amount of work that could be converted from gravitational energy into electrical energy.

2.36 The First Law

This Law is often summarized to "the conservation of energy". Stated equivalently, energy can neither be created nor destroyed. This Law applies to an isolated system that exchanges neither matter nor energy with its environment. The quantity of energy within that system must remain constant. What can change is the quality, or form of the energy (i.e., chemical converted into light, light converted in thermal, mechanical converted into thermal, etc).

The FE provides some insight into this Law. We suggest that the total diversity of the system is perhaps a more fundamental system characteristic that can more completely describe the dynamics of a system's behavior:

$$DTcs = (Dss * De) + Sss$$

As mentioned above, the DTcs expresses all of the potential states that a system can obtain as well as the rates of change a system can undergo. From this raw potential, the initial diversity is constrained by "forces" such as the speed of light, chemical bonds, gravity, etc. Initial diversity is converted into symmetry and continues to be converted as the many-to-one and one-to-one transition mechanism progresses. As the system continues to evolve and the state space (Dss) diminishes, additional diversity can be converted into different De. Upon approaching and reaching equilibrium, as the system's Dss will resolve to unity and the balance of the DTcs must be "absorbed" into the system's De and Sss. We expect that the majority of system's DTcs would be "absorbed", or converted by the MI between system variables that in turn shift the probability distribution of state transitions such that the many-to-one and one-to-one mechanism dominates system behavior. In short, the DTcs does remain constant for isolated systems, it converted however into Dss, De and Sss.

2.37 The Second Law

The Laws described so far deal with WHAT a system can evolve into. For example, the Zero-th Law deals with the final end states of a system's subsystems when they reach thermal equilibrium. The First Law describes the qualitative restrictions of a system's diversity. The Second law deals with HOW systems can evolve. There are two parts to this Law. First, is that diversity differentials between systems that are connected in such a way that they can interact, will seek to achieve equilibrium (differentials resolved to zero). Secondly, diversity flow is always from the system with greater diversity to the system with lower diversity (source to sink). For example, with thermal energy, heat can ONLY flow from the hotter object to the cooler object.

Basically, what this Law describes is the result of many-to-one and one-to-one state transitions: there are no overall system-wide one-to-many transitions in the system's evolution. The symmetry resulting from these many-to-one and one-to-one transitions, can only stay the same or increase, it can never decrease as the system evolves.

Keep in mind, that within the overall system, there can be "local" subsystem behavior that could go counter to this tendency and have a local decrease in symmetry. An example of such a local decrease in symmetry is apparent in the description of the Zero-th Law. In the case of thermal energy, when the subsystem with the lower temperature absorbs thermal energy from the warmer subsystem its symmetry decreased (and the diversity (De) increased). But, despite this local decrease in symmetry, the overall symmetry of the larger "super-system", containing source, sink and intermediary subsystems, increased.

In the course of a system's evolution towards equilibrium, the many-to-one and one-to-one transition patterns cause the Dss to diminish. The Second Law also states that those certain states in the combinatorial space (DTcs) which become unreachable and are therefor "lost" to state space (Dss) can never be "regained" by the Dss. In this sense, there is a characteristic of irreversibility to a system's evolution. Again, there can be local reversibility for subsystems, but the overall super-system Dss can only stay the same or diminish.

2.38 The Third Law

For systems made up of physical matter and undergoing chemical reactions, temperature is an important system property. Recall that to measure temperature is to measure the thermal energy of the system. The temperature of scientific experiments can be measured on a scale called the Kelvin scale. The Kelvin scale is unique in that it is based on the theoretical premise that the thermal energy contained by a system is finite. If some process can extract all of the thermal energy of a system, then the system will reach a temperature of Absolute 0 degrees Kelvin. In this state, a system has lost all of its kinetic energy due to thermal energy. All that remains in such a system are the vibrational patterns that uniquely define each of the elements comprising the system. Referring to the FE,

$$DTcs = (Dss * De) + Sss$$

We find that at Absolute Zero, all of the system's initial DTcs has been converted into symmetry. In particular,

$$DTcs = Sss, \text{ because } Dss = De = 0 \text{ bits/second.}$$

2.39 The Impacts of Intervention

From the above laws it is possible to derive certain corollaries that have particular practical implications to our everyday lives. Below, we introduce two such corollaries that address the impacts of outside intervention on the behavior of a target system - which is the impact of any system operating on another system. Typically, the intervention we are interested in is the desire to obtain some form of benefit (i.e., work) from a system. As a result there is a specialized form of system-to-system interaction whereby one system is being manipulated by another system.

When we get involved in the behavior of a system (especially natural systems), we usually have some objective of benefiting from that system. In economic terms, our objective can be stated as a form of utility gained from manipulating a system. The extent of our utility is determined by the extent of benefit obtained when compared to the extent of effort expended to manipulate the system. Net benefit is the key factor: the amount benefit minus the amount of costs.

Manipulating the system will require some form inputs. For example, effort must be exerted to cultivate fields, obtain seed, and acquire the necessary knowledge to manage such a system, etc - all of which serve to provide the benefit of food. These inputs each have an associated cost. Whether or not the benefit is worth the cost is a vital factor in making these types of decisions. Thus, how we manipulate the system in terms of inputs, is very significant.

From the standpoint of diversity and symmetry, inputs can be viewed as diversity or symmetry "injected" or "imported" into the target system by the operator system. Costs would then be associated with value of the harvested diversity or symmetry. Further, the extent to which any system can be manipulated or operated on is ultimately limited by the diversity of the system doing the operating.

In particular, Conjecture #1 addresses how adjusting the rates of change impacts the system being manipulated. Conjecture #2 addresses the impacts of different strategies for manipulating systems.

2.40 Conjecture #1

This conjecture is derived directly from the First and Second Laws of thermodynamics. Recall that the First Law states that the total quantity of energy contained by a closed system is constant, though, the qualitative forms of the energy can change and DTcs stays constant. The Second Law states that the qualitative changes are unidirectional and result in a reduction in the quantity Dss as all isolated systems evolve towards equilibrium. Conjecture #1 introduces the impact of rates of state change.

For a given system, the composition of energetic forms (De) and system states (Dss) all change with time at certain rates (which are themselves changing with time as well). When the forms of energy in a system are being converted, each conversion can be viewed as one form being converted into one or more different forms. For example, the chemical energy stored in wood can be "released" when the wood is burned. The process of burning the wood converts chemical energy (stored as potential energy in the chemical bonds of the wood fibers) into light and thermal energies. From the standpoint of utility, one form of energy may be more useful than another. The question that arises then is: how can we manipulate the system most efficiently so that it will convert its energy into the form we want?

Referring to the FE again we have:

$$\text{DTcs} = (\text{Dss} * \text{De}) + \text{Sss}.$$

The terms Dss, De and Sss are all based on conditions of the system. To obtain a particular form of utility from the system, we must direct each these terms to achieve that certain configuration of the system, which will give us utility. If we want to achieve a certain state, or set of states, then we must manipulate Dss, De and Sss. By manipulating Dss, we limit the state space to those states that best suit our purposes. By manipulating De, we can manipulate the frequency with which the most desirable states will occur. Finally, by understanding the relationships between the system variables, we can know how to manipulate both Dss and De.

A system left on its own, will evolve to a certain level of symmetry and equilibrium. The process of reaching equilibrium for a given system requires a finite period of time and is called the system's relaxation time. The relaxation time occurs at a certain rate that we can refer to as the relaxation rate.

From the FE we can say that as a system evolves towards equilibrium, the DTcs has a relaxation rate expressed as:

$$\text{RDTcs} : (\text{bits/sec})/\text{sec} = \text{bits/sec}^2 = \text{bits} \bullet \text{sec}^{-2}$$

and for the other terms in the FE that are related to time, the units for the above RDTcs expression are:

RDss : bits/state change
RSss : (bits/sec) / sec

$$\text{bits/sec}^2 = [(\text{bits/state change})*(\text{state changes/sec}^2)] + \text{bits/sec}^2$$

From the standpoint of utility, we are faced with at least two key choices. First, is to determine which state(s) of the system we desire. Once we know the desired state(s) we would need the system to remain in that state, which means we would want the system to stop changing and remain in the desired state. Essentially, this condition is a form of equilibrium. In order to achieve equilibrium, the state space would be to resolve to unity and the symmetry would need to become maximized. The second choice is how long we can wait for the system to finally exist in the desired state(s) that corresponds to the De/dt term above.

To change the Sss/dt term we would need to change the rate of symmetry "build up" in the system we are operating on. In this case we have two basic possibilities. First is to increase the Sss/dt term by applying an understanding of the system's structure. Understanding of the manipulated system's structure requires knowledge of the MI shared between the system's variables as they change in time. Knowledge and understanding are themselves forms of cognitive diversity and symmetry. The operator's cognitive diversity/symmetry would therefor necessarily require sufficient diversity/symmetry to include the diversity of the system they wish to operate on. Costs in this case could be associated with the effort exerted to acquire the necessary understanding. In short, we would seek to operate on the target system such that it is directed towards equilibrium by maximizing the magnitude of the Sss/dt term. The impact of maximizing the Sss/dt term would also impact the Dss/dt term since we know that increasing the rate of symmetry "build-up" will occur with an associated increase in the reduction rate of the state space "shrinkage". As Sss/dt increases, the term Dss/dt would decrease. We could refer to this approach as the Symmetry Maximization or SymmMax. Alternatively, there is the "brute force" approach that requires that the De/dt term be impacted.

Impacting De/dt requires that the rate of state changes per unit time be "accelerated". Diversity in the form of additional changes per unit time would therefore need to be imported and injected into the target system. This can be thought of as "heating" the system up so that it can reach the desired equilibrium state more quickly - thus increasing the relaxation rate. To maximize the relaxation rate can be referred to as MaxRelax. Costs associated with MaxRelax would be related to the relative values of diversity inputs. Often this is the least cost approach because diversity in the form of energy inputs (ie, petroleum, solar, etc) are relatively less expensive than gaining sufficient understanding of the system's symmetry in terms of the MI information shared between system variables.

2.41 Conjecture #2

The first conjecture deals with how fast we can achieve equilibrium in the system being operated on. Conjecture #2 deals with what type of equilibrium we choose to achieve. Basically, there are two broad categories that overlap in certain circumstances.

When a system has reached equilibrium we are faced with the questions of how long do we want it last and how much do want to get out of it. Essentially, the choice we are faced with is to get a little bit for a long time, or get a whole lot right now. This boils down to optimal vs maximal utility yield. Maximal utility yield is based on the notion that the system yields the maximal potential utility it is capable given the condition that it does not exceed its relaxation rate. Think of the two-lake example mentioned previously. Let’s say the upper lake contains one trillion (10^{12}) gallons of water. The surrounding watershed replenishes the upper lake at the “input” rate of 10^9 gallons per year. If we want to "harvest" the energy from the altitude difference between the lakes, the rate of energy harvest is important. We could select a moderate size "pipe" to connect the lakes together such that the rate of water flow through the pipe/generator system was equivalent to the replenishment into the lake: input = outflow of 10^9 gallons/year. From this moderate size system we could generate electricity at a rate corresponding to input equaling outflow.

But, let’s say we want all of the energy real soon. To accomplish this we could increase the diameter of the pipe that connects the lakes as part of a correspondingly larger generating system. In this case, outflow would have to exceed input into the upper lake. Eventually, the upper lake would run dry in which case energy harvest and utility would drop to nothing. Also, relative to moderate size system, the bigger system will cost more to build. Further, the cost of something going wrong with a larger system would probably be much greater than if something went wrong with a smaller system. Thus, there is a greater risk associated with the big system than the smaller system. But, there is often a significant benefit to harvesting the energy sooner. For example, the investment made into the larger system might be re-couped sooner, thus reducing the risk of the investors.

When the final balance of costs and benefits is made, what usually happens is that what is optimal is to harvest utility quickly – get a fast payback on investments made for energy generation. The long-term utility of the moderate size system wold probably yield a greater total amount of energy (the upper lake never running dry for many years), but do so at a slower rate. Often short-term benefits outweigh the long-term benefits – but at a cost.

Introduction: Risk vs. Reward

In this section we will refine our definition of a system, to refer to the most general case.

2.42 Writing, Reading and Mapping

To begin, a brief review: the Universe is a set of variables and systems. A variable is an entity that can exist in various states. A system is defined as an entity that can change the states of, or *write* to, other variables and systems. A system may also be able to map, or *read*, the states of other variables and systems.

Writing (W) to a set of (m) variables and reading (R) n variables, are the most basic defining aspects of system behavior. We can represent reading and writing as Rn and Wm, respectively. Reading occurs when a system is receiving inputs from the Universe or environment. Writing occurs when a system is sending outputs to the environment.

A system's ability to write is governed by its *mapping function*. The mapping function can also have a large impact on how the system behaves when it interacts with the Universe. The mapping function (f), translates inputs (Rn) to outputs (Wm) with a mapping rule.

$$f: Rn \longrightarrow Wm$$, where n and m represent the input and output variables.

A system has a change rate, which can be described as

Change Frequency (F) : number of state changes per unit time F : changes/sec.

A system can have memory, which is the set of internal variables that are readable and writable by only that system. One type of memory is a set of variables that can be read and written to by a single system. This is individual memory. There is group memory also. In this case, the memory belongs to more than one system, which comprises a single composite system.

2.43 Risk and Reward

A system's behavior can sometimes be governed by a mapping function that results in the system maximizing the gains, or payoff, it receives when interacting with the environment (i.e., Universe). Such mapping functions are quite often faced with the dilemma of balancing risk and reward. Risk is always encountered when there is an element of uncertainty involved in making decisions. In the case of a mapping function, this uncertainty can take the form of insufficient input (Rn) to determine the optimal output (ie, output that results in maximal payoff), or a mapping function that does not account for all of the relationships between the variables. The following example illustrates this situation.

2.44 The AMAZING LUCKY NUMBER GAME

Let's say that you encounter the following advertisement. After you read it and think about, what would you do?

The AMAZING LUCKY NUMBER GAME COMPANY will let you play for just **$5 a ticket**.

Each $5 ticket has four scratchable **Lucky Numbers** and five scratchable **Prize Selection Choices**.

For each incredible Prize Selection Choice, you can choose**: Prize #1 or Prize #2** !!!

Prize awards are based on the Prize Selection Choice you make and the matching Lucky Numbers on your ticket.

All Prizes are CASH and completely tax free !!!

Here are the five amazing opportunities you get to play for just $5:

1) GOOD: See all four Lucky Numbers first, and get to make only one prize selection,

2) BETTER: See any three Lucky numbers and make TWO prize selections,

3) BETTER STILL: See any two Lucky Number and make THREE prize selections, or

4) EVEN BETTER: See any one Lucky Number and make FOUR !! amazing prize selections,

5) The BEST EVER: Don't bother looking at any of the Lucky Numbers, just make FIVE !!!! prize selections.

Some prizes are **Booby Prizes**, which means that if you should unfortunately choose the wrong prize, then **YOU** have to pay the LUCKY NUMBER COMPANY up to $5,000,000[1]

But, because we really want you to have the VERY BEST chances to win, we will **even show you** all of the possible Lucky Numbers and the choice of prizes you get to win **before** you ever have to play !!!

If the prize value is **positive**, then its the cash **YOU WIN**,

If **negative**, oh well...(refer to The Fine Print)

Plus, we even make sure that there are equal quantities of each ticket available at all stores all the time!!!

Lucky Numbers	**Tax Free Cash Prize Selection Choice Value #1**	**#2**
0000	1,000,000	10,000
0001	10,000	-- 500,000
0010	10,000	10,000
0011	1,000,000	10,000
0100	10,000	-- 5,000,000
0101	-- 500,000	-- 500,000
0110	10,000	1,000,000
0111	-- 500,000	10,000
1000	10,000	-- 500,000
1001	10,000	-- 500,000
1010	-- 500,000	10,000
1011	1,000,000	10,000
1100	10,000	10,000
1101	-- 5,000,000	1,000,000
1110	-- 500,000	1,000,000
1111	10,000	10,000

LUCKY NUMBERS is played seven days a week. Here is the schedule:

Sunday: out of respect .... a one ticket limit per player
Monday: mellow Monday............ten ticket limit
Tuesday: this week is OK so far..twenty ticket limit
Wednesday: hump day.............. thirty ticket limit
Thursday: almost over............. fifty ticket limit
Friday: yea, TGIF........... one hundred ticket limit
Saturday: lets party!!!... five hundred ticket limit

SO, CASH IN YOUR PAYCHECKS NOW !!!! BUY YOUR TICKETS WHILE SUPPLIES LAST !!!

The Fine Print.

[1]THE LUCKY NUMBER Company's representatives, the Brutali brothers (Guido "the Razor" and his little brother, Luigi "the Hammer") will collect a onetime, cash payment within 24 hours of selecting any Booby Prize.

2.45 Post-hype Debriefing

OK, so after the adrenaline level in your blood stream subsides and your hands stop shaking with the feverish excitement of unbridled avarice...

1. Would you really want to play this game?
2. If not, how come?
3. If yes, then how would you play the game - especially if you intend to steer clear of Guido and Luigi?
4. Would you have different strategies for each day of the week?
5. How does your risk tolerance impact the type of start strategy you would employ, assuming you wanted to play the game in the first place?

2.46 Analysis

One way to analyze this GAME is to evaluate the various ways you could play each ticket you buy. Another angle we can analyze is how to play different numbers of tickets per day (which is determined by the Schedule - from a single ticket to as many as 500 tickets in a single day).

However, the crucial aspect to this GAME is to first determine what your goals are. If you just want to maximize your gains and take no chances (i.e., minimize uncertainty), then you will choose a strategy that maximizes the number of LUCKY NUMBERs you get to read. By maximizing the number LUCKY NUMBERs you get to see, you are able to clearly see which Prize Selection Choice yields the maximum payoff. But, what if you want to make more Prize Selections per ticket?

2.47 How to play a Ticket

In order to make more prize selections per ticket, you are required to give up reading one LUCKY NUMBER for each additional prize selection you can make. What this means is that you give up some certainty for more opportunities to win prizes. The problem of course, is that some of the prizes involve catastrophic negative payoffs. The constraints imposed by the GAME rules can be looked at this way:

See 4, make 1 choice : 4 + 1 = 5
See 3, make 2 choices : 3 + 2 = 5
See 2, make 3 choices : 2 + 3 = 5
See 1, make 4 choices : 1 +4 = 5
See 0, make 5 choices : 0 + 5 = 5

Notice that in all cases, the number of LUCKY NUMBERS that you can see, or read, plus the number of PRIZE SELECTION CHOICES you can make, or write, always equals five.

Since each ticket costs (only...) $5, the sum of five could be viewed as the sum of five separate one dollar payments - one dollar for each LUCKY NUMBER you get to read and one dollar for each PRIZE SELECTION CHOICE you can write. Given this limit of five dollars, your strategy options become clearly delineated.

1. Strategy #1: Read all four numbers and have maximum information, write once for minimal chances at winning more payoffs
2. Strategy #2: Read any three numbers, give up 25% of available information, and write twice for a 100% increase in chances to win additional payoff.
3. Strategy #3: Read any two numbers, give up 50% of available information, and write three times for a 50% increase in chances to win additional payoffs.
4. Strategy #4: Read any single number, give up 75% of available information, and write four times to gain a 33% increase in chances to win additional payoffs.
5. Strategy #5: Do not read any numbers, give up 100% of available information, and write five times to gain a 20% increase in chances to win additional payoffs.

2.48 Risk Tolerance

By now the trade off between risk and reward should be a little clearer. The bottleneck is always that you have a limit on how much you can "spend" for reading and writing: the more you read, the less you can write, and visa versa. The less you read, the more uncertainty you encounter, but you trade certainty for more chances to win prizes. The catch of course, is that having more chances to win prizes, with less information, also means more chances to "win" the negative payoffs. To a certain extent, the degree of uncertainty, or risk, you can tolerate is a subjective decision, based on such considerations as:

1) What do you have to lose? If you really do not care that you might lose the BIG $5,000,000 BOOBY PRIZE, because you could easily pay off Guido and Luigi, then what the heck? In this case, your risk tolerance is high and could be a symptom of entertainment addiction.

2) On the other hand, you might have only a maximum of a few hundred dollars to your name, *and* a horrible fear of death by torture. With these considerations, you would be risk averse – your tolerance of uncertainty is zero since you risk a potential loss from which you could not recover, i.e., death. Given these varying degrees of risk tolerance (from the very high to the very low), your choices of strategies will be shaped accordingly.

2.49 Different Start Strategies

Once you have determined how much risk, or uncertainty you want, you need to consider the impact of your start strategy. If you were to decide, for example, that all you can afford is to just buy one ticket, and you do not want any risk, then Strategy #1 (refer above) is the one for you. If you can afford the uncertainty, then Strategies #2 - #5 would be likely choices.

2.50 Strategies for how to play the GAME each day of the week

The big difference between the each day of the week is the number of times you can write which, is ultimately limited by the number of tickets you can buy. If you look carefully at **Table 2.12**, presented again below, notice that for both Prize Selections #1 and #2, there are an equal number of chances to be awarded each of categories of cash prize, namely the $1,000,000 or the $10,000 gains or, the -$500,000 or -$5,000,000 losses.

Here is a summary of each category:

Prize Category	Occurrences as Prize Selection #1	#2
10,000	8	8
1,000,000	3	3
-500,000	4	4
-5,000,000	1	1

Table 2.12: Summary of Cash Prizes for each Lucky Number

Lucky Numbers	Prize Selection Choice Value #1	#2
0000	1,000,000	10,000
0001	10,000	**-500,000**
0010	10,000	10,000
0011	1,000,000	10,000
0100	10,000	**-5,000,000**
0101	**-500,000**	**-500,000**
0110	10,000	1,000,000
0111	**-500,000**	10,000
1000	10,000	**-500,000**
1001	10,000	**-500,000**

1010	**-500,000**	10,000
1011	1,000,000	10,000
1100	10,000	10,000
1101	**-5,000,000**	1,000,000
1110	**-500,000**	1,000,000
1111	10,000	10,000

2.51 Expected Gains and the Importance of Frequency

The question to consider now is focused on the number of tickets you can buy each day. In other words, how is buying just one ticket different from buying 500?

The basic idea centers on our assumptions of randomness. Previously, we have discussed randomness and used the expression $P(N) = 1/n$ to describe the probability of a random event occurring (where N is a random event and n is the number of N that can occur).

We will also assume that the LUCKY NUMBER COMPANY is honest enough to really and truly have equal quantities of tickets available all the time and that the chances of buying any ticket are equal.

If a random event occurs many times, or at a high frequency, then we would expect the number of times each event occurs to become very similar over time. Think of throwing a die only once, vs 10 times, vs 100 times and so on. Eventually the frequency of any face occurring will approach 1/6 of the time. The same applies to which Lucky Number you could find on your ticket.

The chance of winning any prize is 1 in 32. When you take the product of probability and the associated gain, you have something *Expected Gain* or E(G). The E(G) for Lucky Numbers is summarized below in **Table 2.13**. In **Table 2.14**, we summarize the E(G) for each ticket. These tables also show that the overall EG for the entire game.

Table 2.13: Expected Gain for each Prize Selection

Lucky Number	Prizes #1	Prizes #2	Chance	Expected Gain #1	Expected Gain #2
0000	1,000,000.00	10,000.00	1/32	31,250.00	312.50
0001	10,000.00	(500,000.00)	1/32	312.50	(15,625.00)
0010	10,000.00	10,000.00	1/32	312.50	312.50
0011	1,000,000.00	10,000.00	1/32	31,250.00	312.50
0100	10,000.00	(5,000,000.00)	1/32	312.50	(156,250.00)
0101	(500,000.00)	(500,000.00)	1/32	(15,625.00)	(15,625.00)
0110	10,000.00	1,000,000.00	1/32	312.50	31,250.00
0111	(500,000.00)	10,000.00	1/32	(15,625.00)	312.50
1000	10,000.00	(500,000.00)	1/32	312.50	(15,625.00)
1001	10,000.00	(500,000.00)	1/32	312.50	(15,625.00)
1010	(500,000.00)	10,000.00	1/32	(15,625.00)	312.50
1011	1,000,000.00	10,000.00	1/32	31,250.00	312.50
1100	10,000.00	10,000.00	1/32	312.50	312.50
1101	(5,000,000.00)	1,000,000.00	1/32	(156,250.00)	31,250.00
1110	(500,000.00)	1,000,000.00	1/32	(15,625.00)	31,250.00
1111	10,000.00	10,000.00	1/32	312.50	312.50
Total Expected Gain				**(122,500.00)**	**(122,500.00)**

Table 2.14: Total Expected Gain for Entire System

Combined Chance	Prizes	Combined Expected Gain
1/2	$ 10,000	$ 5,000
3/16	$ 1,000,000	$ 187,500
1/4	$ (500,000)	$ (125,000)
1/16	$ (5,000,000)	$ (312,500)
	Total Expected Gain	**$ (245,000)**

Exercise:

How can you (wisely?) trade off reading information for extra chances to win?

Which Lucky Number has the greatest value for risk reduction? – for gain maximization?

2.52 The Information/Energy Chart: A Key to Understanding How Systems Read and Write to the Environment

What is a Continuum?

For this discussion we will use as the operational definition for a continuum: *a progressive gradient transition between polar opposites*. The transition between polar opposites can be viewed as an evolution and is, conceptually at least, bi-directional.

2.53 Information - Energy Continuums

We seek to construct a conceptual framework that integrates general systems theory, information theory and thermodynamics to answer the question: "How is my knowledge of an observed system (S) changing with time? " Or equivalently, "how does the ability of O, to read and write to S, change through time"?

The unifying conceptual theme for this framework is a continuum that begins with *diversity* and progresses to *symmetry*. The term diversity corresponds to a set of system properties that, in the extreme, correspond to an unknown number of random system states. Symmetry, at its extreme, corresponds to a known, deterministic, state space.

To accomplish this integration, the conceptual framework both differentiates and traces the parallel and multi-dimensional evolution of potential understanding for S by O as well as the potential evolution of S itself, independent of any observers. We suggest that the parallel evolution of these two processes is often linked because, what may appear as randomness or chaotic behavior in S is actually the lack of understanding by O for an otherwise deterministic S. For systematic completeness, the evolution begins with both the total ignorance of O for S and the total diversity S. The evolution ends with the complete understanding of O for S and complete symmetry for S. Finally, we will argue that this movement towards symmetry is a direct analogue of the Second Law of Thermodynamics.

2.54 A Top-Down Flow, and Vertical Axis of, Evolution

The Diversity/Symmetry Continuum can be viewed as a vertically oriented axis. All those system characteristics associated with complete diversity are located at the top of the axis and can transition, or evolve, towards all those system characteristics associated with complete symmetry at the bottom of the axis: complete diversity at the top and complete symmetry at the bottom. These axes form a graph that can be referred to as the *Information/Energy Chart*, or *I/E Chart*, for short. Refer to **Figure 2.11**.

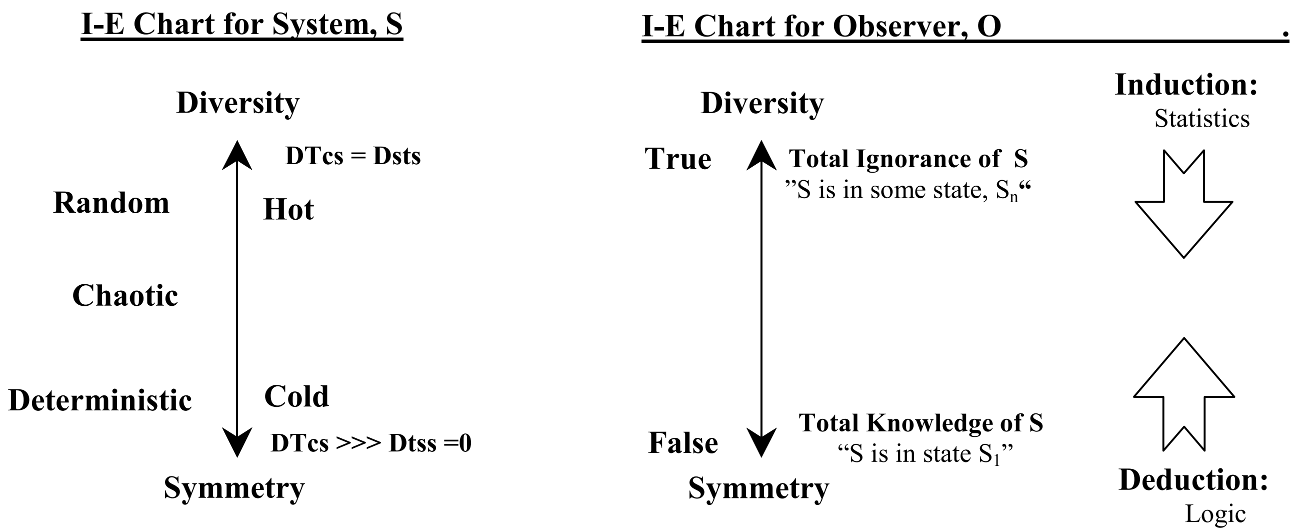


Figure 2.11: The Information-Energy Chart

Information/Energy can be thought of as the amount of Diversity/Symmetry being read from and/or written to S by O, it is some pattern that the two systems share, have in common, and is similar, thus a "bond" of symmetry between them. The process of transferring information and energy is called *communication.*

Information can be thought of as the symmetry that is communicated between systems. Energy is diversity. More specifically, energy is the frequency component of diversity - the number of state changes per unit time. Energy has many forms depending on the frequency and type of system. Energy can be transferred between systems. Energy transfer is also a form of communication. Both the transfer of symmetry and diversity are governed by the Laws of thermodynamics - especially the Second.

2.55 The System (S) and the Observer (O)

The I/E continuum represents two parallel processes: the evolution of at least two systems. For this discussion, we will divide the Universe of variables and systems into three parts: the system being observed, S, and the system doing the observing, O, and a shared environment, E.

Initially, we can assume that S and O act independently of each other. S is doing its own thing: changing or not, reading and writing to its environment at various degrees through time. O is a separate and different system from S. The system O is special, however, in the sense that O is reading S with the intent of eventually being able to write to S.

2.56 The I/E Chart and the Second Law of Thermodynamics

Progressing from the Top to the Bottom of the I/E Chart:

1. All isolated systems move to equilibrium: from free to constrained, where free corresponds to a large state space and constrained corresponds to a small state space.
2. Path of least resistance (PLR) - as a system gets organized, its mapping rule will seek those states that use the least amount of potential diversity per state change, where potential diversity is Dp and Sss = Dcs - [Dss * (Dk + Dp)].
3. Thermal energy flows from hot to cold
4. De (the frequency, ***f***) will seek to diffuse itself - flow from areas of high ***f*** concentration to areas of low ***f*** concentration - until the common energy "field" has uniform distribution of ***f*** concentration.
5. Closed and open systems can seek to expand the diversity of their DTcs by "reaching out" into E to obtain more DTcs and De.

2.57 From Ignorance to Knowing and the Veil of Chaos

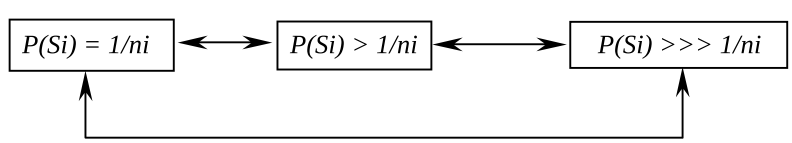
Since much of this discussion is based on the point of view that belongs to O, we can begin with O being completely unable to even read S, let alone being able to write to S. We thus begin with a condition where O is completely ignorant of S.

Now imagine, that by some series of events, O becomes aware somehow of the presence of S (perhaps indirectly via some type of interaction that O detects between a known system, Sk, and the unknown S). Let’s say that once O "suspects" the existence of S, it attempts to read S. At first, what O reads may not "mean anything" to O. The inability of O to make any sense of what it reads from S is part of the definition of ignorance. Often, when O does some reading of an S and O is not able make any sense of S, the behavior of S is sometimes referred to as displaying (or having) chaotic or random behavior.

Once O has had sufficient opportunity to read S, and also "learned" how to "understand" S, then what was initially chaotic behavior can transform into behavior with recognizable patterns. O’s understanding of S may at most be very limited. In part, the limiting factor that governs O’s ability to understand S is O’s own degree of diversity. Further, the patterns of S that are "learned" by O may not necessarily be characterized by simple behavior and may in fact be extremely complex (more on the subject of complexity later...). In other words, for O to fully understand S, O must be at least as complex as S. Thus, the total diversity of O must be equal to, or greater than the total diversity of S: $O_{DTcs} >= S_{DTcs}$

2.58 From Randomness to Determinism

As S evolves, the probability distributions generated by its inter-state mapping function can become more deterministic. This would be because S is progressing towards equilibrium. At the top of the I/E is S would be completely random, at the bottom it is completely deterministic. It is also possible for S to achieve equilibrium and then change into a state of dis-equilibrium. S could also cycle between the states of equilibrium and dis-equilibrium. This is especially true of systems that are able to “reach out” into their environments to acquire additional diversity, thus expanding the potential of their combinatorial space, DTcs.



2.59 From True to False

In the context of the I/E Chart, true and false, in the strict sense of *propositional logic*, are also parts of the continuum. The continuum of true-false belongs primarily to the Observer’s understanding of S. Recall that O wishes to understand S in terms of accurately knowing the true state of S. For S, there are two classes of statements about its actual state: those statements that are true and those that are false. Each of these two classes can be viewed as a set of statements. Let the set of statements about the state of S that are true be: $T_s = \{s_1 \ldots s_n\}$. For simplicity, let the set of all statements about the state of S that are false be expressed as an empty set because they contain "no truth" about S: $F_s = \{Ø\}$

For O, the situation is a little bit different. O's understanding of S can be expressed as the intersection of sets T_s and F_s:

$$T_s \cap F_s = \{Ø, s_1 \ldots s_n\}.$$

Refer **Figure 2.12**. Notice that the member of set T_s U F_s which belongs to *all subsets* that are true is {Ø}.

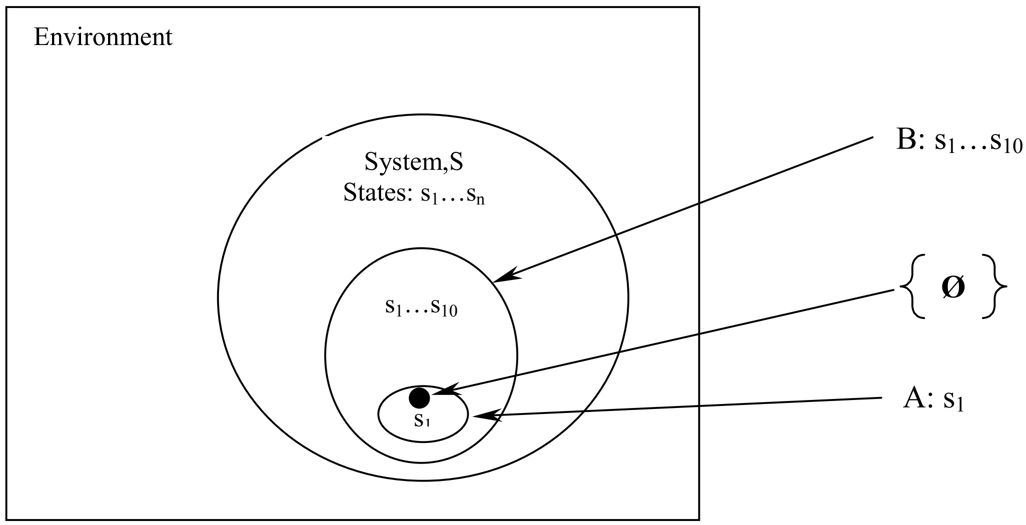


Figure 2.12: Observer's Understanding for the States of System, S

In the beginning of the Observer's understanding of S, she knows nothing. The Observer could say that "S is in a state: $\{Ø, s_1 \ldots s_n\}$". This does not say too much because it does not specify a single specific state of S. For example, imagine a meteorologist saying "…the air *will have* a temperature today…" This statement obviously does not contain too much useful information. What we want to know is the percent chance of the air being within a specific range of temperatures, or even a certain specific temperature. Compare the information content of the previous statement to: "…there is a 90% chance that the air temperature today will be between 55 and 65 degrees…"

As O learns more about S, O will be able to "narrow down" the number of possibilities that have a reasonable probability of being true. For example, O could say that S is in one of the states that belong to the subset of possible states $s_1 \ldots s_{10.}$ The process of O gaining better understanding of S can continue to evolve in the form of O being able to say that at a particular point in time S is in state $s_{1.}$

This process of gaining understanding results in O having to become more specific about which state S is in. Thus, if S is in state s_1 , then the statement: "S is in one of the states belonging to the subset $s_1 \ldots s_{10}$" has less information content than the statement: "S is in the state s_1". Thus, the process of increasing specificity reduces the diversity of O's understanding of S. As O gains better understanding, O progresses down the I/E Chart.

The process of gaining understanding for S also involves a process of subset development. Again, refer to Figure #10. The subset $\{Ø, s_1 \ldots s_{10}\}$ belongs to the larger set $\{Ø, s_1 \ldots s_n\}$. We can say that $\{Ø, s_1 \ldots s_{10}\}$ *subsumes* $\{Ø, s_1 \ldots s_n\}$. As O's understanding of S progresses, the set of true statements can finally resolve to $\{Ø, s_1\}$ which belongs to the larger set $\{Ø, s_1 \ldots s_{10}\}$. Now we can say that $\{Ø, s_1\}$ belongs to, or subsumes, the larger set $\{Ø, s_1 \ldots s_{10}\}$. Notice that the one member belonging to all subsets is {Ø}. Thus, the limit of symmetry exists in the one subset that subsumes all others: {Ø}. The set {Ø}, which is the empty or null set, is also the set of all statements about S that are false.

The notion behind the progression from true-to-false is demonstrated in the use of a *truth table*. With the truth table we can test the truth-value of a statement fitting the template of "A implies B". Again, we refer to both **Figure 2.12** and **Table 2.15**. In **Table 2.15**, notice that A and B represent specific statement about S. The statement A specifies a single state of S, namely that S exists in the state s_1. Statement B has less information content than A from the standpoint that it defines a subset of states that S could be in. One of the most important distinctions to keep track of is the relationship between A and B. Note that in all cases, A subsumes B. Also, notice that there are four cases: the first three are rather straightforward, the last case is lots of fun…

Table 2.15: Truth Table for A Implies B: A⟶ B

A: S is in state s_1
B: S is in state $s_1..s_{10}$.

A	B	A ⟶ B
T	T	T
F	T	T
F	F	T
T	F	F

The first case says that "If A and B are true, then A⟶B is true". This makes sense because if S is in state $\{s_1\}$ then it is also in $\{s_1..s_{10}\}$ and the notion of A implies B fits.

The second case says that "If A is false and B is true, then A⟶ B is true". This case makes sense because S *could* be in some other state, say $\{s_4\}$, instead of state $\{s_1\}$ which means that S is still "contained" in the set of states $\{s_1..s_{10}\}$ and "A implies B" still makes sense.

The third case: If A and B are both false, then A⟶ B is true", makes sense because S could really be in state $\{s_{110}\}$. In this case, S is neither in state s_1 nor in one of the states $\{s_1..s_{10}\}$. So both statements A and B would be false. Thus, "A implies B is true" makes sense when both A and B are false.

Finally the forth case says that: "If A is true and B is false, then A⟶ B is false". To better understand this case, lets say that the reality of S is that it can only exist in states $\{s_a…s_z\}$. In other words, statements A and B are essentially *different* systems: A: $\{s_1\}$ and B: $\{s_a…s_z\}$. Thus, if the reality of S is not $\{s_1..s_{10}\}$, then saying that S can exist in $\{s_1..s_{10}\}$ is false. For the statement A to be true, it would have to be referring to a different S, namely $\{s_a…s_z\}$. Thus, to say that A implies B would be false.

The progression of these four cases also corresponds directly to the progression *down* the I/E Chart. The top of the I/E Chart corresponds to the first case of T T T . The bottom extreme of the I/E Chart corresponds to the last case of T F F where we could say that the system we were originally investigating has changes into another system, or has "broken".

2.60 Induction, Deduction and the I/E Chart

There are many other dimensions to the I/E chart that are interesting. The last one that we cover in this discussion follows the thread of diversity at the top of the chart referring to the generality of O's knowledge of S. As O learns more about S, O's knowledge becomes more specific. The terms *deduction* and *induction* refer to forms of reasoning that basically go in opposite directions along the I/E chart. Going down the I/E corresponds to inductive reasoning and deduction corresponds to going up the chart.

To begin, let's start with inductive reasoning – we start at the top of the I/E chart. Referring to Table 15, inductive reasoning corresponds to B ⟶ A. The idea here is that I want to know the exact state of S. But, all I am certain about is that S is in one of the states $\{s_1..s_{10}\}$. Therefore, for me to become more specific and progress from what I know for sure ("in general, S is in one of the states $\{s_1..s_{10}\}$") to what I am not certain about ("S is in state $\{s_1\}$"), then I must apply inductive reasoning. In the process I gain specificty but lose certainty – and in this sense inductive reasoning contains an element of risk.

One of the most common forms of inductive reasoning is found in statistics. Statistics can be viewed as an analytical process that measures a charactieristic of a population and then makes a conclusion about an individual from the same population. For example, say we wanted to know the height of each individual in a very large group of people (the "population"). Next, we are faced with the common problem of not being able to measure the height of *every* individual in the group. So, we take a *sample.* The sample is a subgroup which is, ideally, a representative of the overall population. Once we have measurements from the sample, we can calculate an *average.* With the average, we can make some HUGE assumptions, then apply inductive reasoning, and say that the height of each individual in the population *is* that average (often there is a certain degree of "confidence" given to the degree of accuracy of the average). Overall, what is accomplished here is going from general knowledge about a system, to specific knowledge via assumptions that permit going from the general to specific. Refer to **Figure 2.16**.

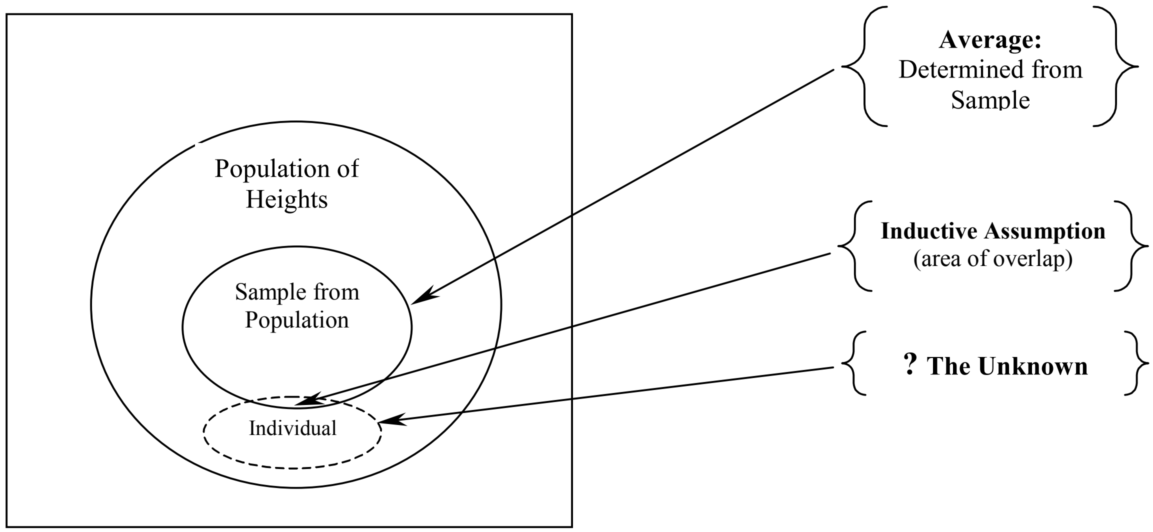


Figure 2.16: Example of Inductive Reasoning: Statistical Determination of Individual Height from Population Sample

On the flip side of the I/E chart, is deductive reasoning. In this process, we lose specificity. We start at the bottom of the IE chart knowing that S is in state $\{s_1\}$ and can then say that S is also in $\{s_1..s_{10}\}$. Referring to **Figure 2.12** and **Table 2.15**, deduction coressponds to $A \longrightarrow B$.

Exercise:

Is the following statement true? Why, or why not? Discuss.

> *If $A \longrightarrow B$ is true, then the probability of event B occurring, P(B), is less than or equal to the probability of event A, P(A), occurring, or P(B)<= P(A).*

Ω