WiFi Access Point Pricing as a Dynamic Game

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Abstract— **We study the economic interests of a wireless access point owner and his paying client, and model their interaction as a dynamic game. The key feature of this game is that the players have asymmetric information – the client knows more than the access provider. We find that if a client has a web browser utility function (a temporal utility function that grows linearly), it is a Nash equilibrium for the provider to charge the client a constant price per unit time. On the other hand, if the client has a file transferor utility function (a utility function that is a step function), the client would be unwilling to pay until the final time slot of the file transfer. We also study an expanded game where an access point sells to a reseller, which in turn sells to a mobile client and show that if the client has a web browser utility function, that constant price is a Nash equilibrium of the three player game. Finally, we study a two player game in which the access point does not know whether he faces a web browser or file transferor type client, and show conditions for which it is not a Nash equilibrium for the access point to maintain a constant price.**

Index Terms— **Game Theory, WiFi Pricing, Network Pricing.**

I. INTRODUCTION

Today there is a large and growing number of wireless access points deployed by homes and businesses for private LANs. Many of these access points could potentially be used to provide Internet access to users from the general public that lie or are passing within communication range of the access point. However, owners of private WiFi networks often choose to encrypt their networks to prevent outsiders from accessing them. Without a mechanism for a potential client to compensate the owner of the network, the network owner has no reason to accept the increased network traffic and security risk that would come from allowing the public to access his network.

If it were possible to incentivize owners of existing private wireless access points to open their networks to

the public, as well as incentivize people and institutions to deploy access points where there are gaps in coverage, the result might be nearly ubiquitous WiFi coverage. In contrast to cellular phone networks deployed by a few large providers, this ubiquitous access network would be deployed by thousands, perhaps millions, of autonomous self-interested agents.

A. Alternative Charging Models

The simplest way for a client to compensate an access point owner would be for the client to just pay the access point directly. We refer to this as a Peer to Peer (P2P) model, because it does not involve a 3rd party. Other models are possible. One model that is becoming increasingly popular is the aggregator model. In the model, the deploying business partners with an aggregator franchise, such as Boingo [19]. The aggregator attaches its brand name to hot spots and ensures that a consistent product is offered among the hot spots deployed by different businesses. The aggregator also handles the billing for the service, and can offer the user subscription billing plans that apply to all of the branded hot spots. The aggregator collects the revenue from the client, and then redistributes some of the revenue to the deploying business partners. This model will likely continue to grow in popularity, but because it requires a third party to monitor that each access point adhere to the standards of the aggregator brand name, it is not clear whether this model could scale to millions of access points. Perhaps, an aggregator model more like that used by online auction sites would be more scalable; a model in which clients can view a reputation rating based on past clients feedback, but where the aggregator does not guarantee the trustworthiness of a particular access point. In this work we will not study the aggregator model further, but instead we will focus on studying the properties and viability of a P2P model involving the two principle parties, access point and client, with minimal or no third party involvement.

B. Making the P2P Model Viable

Though the P2P model has the potential of being more scalable than the aggregator model, there are a number of challenges in making a P2P model viable. In many

cases a client and access point may not know each other's identity, and may not be able to trust each other to carry out their side of a transaction. To understand the problem, imagine a scheme where the client pays for her entire session in one lump payment. In a scheme where the client pays the access point in advance, or pre-pay scheme, a malicious access point might accept payment and then fail to deliver service. In a post-pay scheme, a client may fail to make a promised payment after receiving service. In fact, if we imagine that the access point and client are players in a game, and are trying to maximize their reward from this single transaction, a client *should* try to obtain service without paying, and a access point *should* try to take the client's money without giving him any service. Therefore, we must take care in structuring the game in a way that deters the players from cheating. Yet at the same time, we would like to avoid introducing a third party enforcement agent to the game. In an implementation, an enforcement agent would probably have to be centralized, and thus might limit scalability.

One possibility would be for the client to pay the access point in small amounts over the duration of the session. The intuition justifying this scheme is that a access point will want to play "fair", lest it be punished by being denied payments in the future, and a client will want to keep paying throughout the session to ensure that its service is not cutoff. We must also be concerned with how the access point changes its access price over the duration of a session. Will the access point entice the client to connect with a low price in the beginning, and then later threaten to cutoff the client's file transfer unless she agrees to pay a new higher price rate? Will the client refuse to connect to a access point out of fear that the price will be unstable during the duration of a session? These are the kinds of question we hope to address in this work.

C. The Problem of Contract Enforcement in a P2P Model

One idea of how to deal with client or access point misbehavior would be to have a contract between access point and client. If either party deviated from the terms of the contract, then the other would take up the issue with an enforcement agent and seek that the offender be penalized. The threat of the penalty would keep both parties honest, making it very rare that anyone actually need to contact the enforcement agent, and thus making it possible to scale this model to a very large number of contracts. The problem with this model is that a client would have an interest in falsely accusing the access

point of not delivering service, and without a potentially expensive connection monitoring scheme, the enforcement agent would have no way of knowing whether the client's accusation were valid. For this reason, we seek to avoid the need for contracts in the P2P charging model.

D. Possible Payment Architecture

The model we are envisioning assumes that the client pays the access point in small payments over the course of the session. Unfortunately, most electronic payment schemes in common use today have a relatively large transaction overhead. If the length of a time slot were only on the order of a few minutes or less, it is likely that the transaction overhead would be comparable in size to the size of the payment itself. Clearly, such a high payment overhead would be prohibitive. Much work has been done in the area of making small payments, or micropayments, with a minimum of overhead. One scheme in particular appears to be promising in this context. The PayWord scheme, proposed by Rivest and Shamir, [1], [2] makes it possible for the payee, in our case the access point, to aggregate many payments from a client into a single, larger payment. The scheme works on the principle of pay chains. In the brief description that follows, we assume the reader is familiar with the ideas of digital signatures and one way functions. For a review of these concepts, the reader can consult [3].

Figure 1 illustrates how the PayWord scheme can be used in WiFi access point charging. Prior to beginning a session, a client would compute what the authors of [1] calls an "H-chain" by repeated evaluation of a one-way function H. Specifically, the "H-chain" consists of values $x_0, x_1, ..., x_T$ where $x_{t-1} = H(x_t)$ for $t = 1, 2, ..., T$.

A client begins a session by passing the access point the root of the chain x_0 that has been signed with the client's digital signature. In each subsequent time slot, the client makes another payment by passing the next consecutive value of the H-chain to the access point. When the access point is ready to deposit the micropayments, the access point can combine them into single deposit by passing to the bank the client's digitally signed x_0 and the value, x_T. The bank can verify the authenticity of x_0 by its digital signature, and can verify that T micropayments were made by verifying that $H^T(x_T) = x_0$. The most important property of this payment scheme, is that it does not involve a third party for each micropayment. The access point need only contact a certificate authority at the beginning of the session to verify the authenticity of the client's digital signature of x_0. The access point can independently verify the authenticity of successive micropayments by verifying $x_t = H(x_{t+1})$.
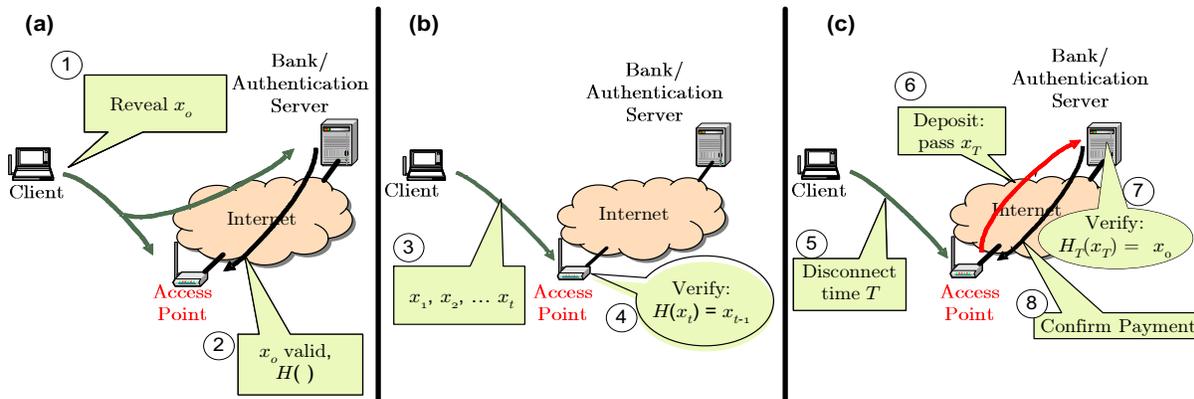
Fig. 1. Illustration of payment architecture. Note that the authentication server is only involved at the beginning and end slots of a session. Steps: (a) The client reveals root of pay chain, x_0; the authentication server verifies that validity of x_0. (b) The client reveals x_1, x_2, ... as time slots pass. (c) The access point deposits his payments by passing x_T to authentication server.

E. Security Issues

Security is another critical issue that the P2P model of WiFi access faces. A Client needs to know that the access point owner is not eavesdropping on her traffic, or spoofing the websites that she is trying to reach. Similarly,an access point needs to know that a client cannot use her access to launch an attack on his network. We believe that most of these concerns can be addressed by existing techniques. For example, the client could use a secure tunnel to a home agent on the client's home network to prevent the access point from eavesdropping or spoofing. The access point could use a firewall to prevent the client's traffic from going beyond the access point and gateway router into the rest of the access point owner's network. A harder problem is preventing the client from overusing the access point owner's link to the Internet. Though the access point might rate limit the client's incoming traffic to prevent the Internet uplink from being overused, the client might overuse the downlink, or even deliberately overuse the downlink in a form of denial of service attack against the access point owner's network. One solution might be to measure the client's downlink usage, and terminate her service if her usage is excessive. For the rest of this paper we will assume that these security issues are solvable and instead focus on the strategic interactions between an access point that is free to set and change prices over time, and a client seeking to maximize her utility.

F. Previous Work

Our model is that of an single access point, or seller, and a single buyer, the client. In the economics literature, a model with a single buyer and seller is called a "bilateral monopoly." The static case of bilateral monopoly, the case where repeated transactions over time are not explicitly modelled, is well known and is found in economics text books [4]. What differentiates our model from the standard, static bilateral monopoly model is that we model our situation as a multi period dynamic game, where players have to consider the effects of their actions on the future as well as the present. There has been some other work in studying bilateral monopoly situations using dynamic games. For example, Vincent looks at a bilateral monopoly in which buyers may either be low or high valuation types, while the seller tries to learn the buyer's type from his behavior over the duration of the game [5]. The literature in bargaining, where two parties negotiate for a share of a surplus over multiple periods, is also closely related to dynamic, bilateral monopoly models [6], [7]. However, as far as we are aware, there is no prior work in either dynamic game models of bilateral monopoly or bargaining that capture the key features of the models we develop in this paper.

The networking community has also focused a lot of attention on pricing ideas in recent years. For example, many researchers have looked at congestion pricing, where resources, often network links, are priced according to how heavily they are being used, so that users are incentivized to redirect traffic to less congested, less costly links, or to lower the bit rates of their flows [8], [9]. At this point, our models do not look at how prices can be used to reduce congestion, but instead look at the strategic relationship between a buyer and

seller. Network researchers have also looked at using mechanism design principles to get users to bid their true utilities for network services [10], [11], [12]. The mechanism design methodology assumes a "principal" player who collets the player's bids and is trustworthy. In contrast, our game model assumes that the access point and client face each other in the game without oversight of a neutral principal. Other networking researchers have applied game theory to congestion pricing [13], and splitting revenue between multiple providers [14], but none of these models capture all of the aspects of the structure of the Wi-Fi pricing situation we study in this work.

G. Overview

In Section II we introduce our basic two-player game model. In Section III we discuss an instance of the basic model that we call the web browsing model, and show that the provider or access point should charge his client a constant price. We also show how the web browsing model can be extended to a multi-hop scenario. In Section IV we introduce the file transferor model, and show that if the access point has an a priori bound on the possible length of the client's file, the client's dominant strategy is to refuse any price greater than zero until the final time slot of her file transfer. The access point, in turn, charges a price of zero, until a critical slot t^* is reached, and then tries to charge for the whole session in one shot. In Section V, we study a model in which the access point is unsure whether he faces a file transferor or web browser client. In Section V-A, we investigate what happens when a file transferor type client's file length has no a priori bound. Section VI summarizes and assesses our results.

II. BASIC MODEL

We can formulate the interaction between a access point and paying client using a simple two-player game model. The game progresses in discrete time slots or "periods." At the beginning of the first time slot, the access point proposes an access price, p_1 for access during the first time slot. The client can either accept the price and connect, or reject the price and not connect. If the price is rejected, the game ends and both client and access point receive zero payoff.

In general, the access point offers connectivity at the beginning of time slot t at price p_t. The game ends the first time the client rejects the access point's proposal.

The client's utility function $F(T, \tau)$ is a function of the number T of time slots the client chooses to connect and a parameter τ which we call the client's intended session length. T is a decision variable; it is a function of the actions the client takes in each time slot, specifically the number of slots the client chooses to connect. In contrast τ is a *type* variable that specifies the maximum time the client would be interested in connecting. For instance if the client were sitting on a park bench with a laptop and needed to leave the bench in 30 minutes, and the slot time were 1 minute, then τ would have a value of 30. The client does not choose τ in the game, but instead it is determined for the client by outside circumstances. The client gets to know the value of τ at the beginning of the game while the access point only knows its probability distribution.

The client's net payoff is $F(T, \tau) - \sum_{t=1}^{T} p_t$, while the access point's net payoff is $\sum_{t=1}^{T} p_t$. The underlying assumption is that the access point's marginal cost to provide the service to the client is negligible.

We study the Nash equilibria of this game, under different assumptions of the structure of the utility function $F(T, \tau)$. We assume the reader has some knowledge of game theory in the discussion that follows. A reference for the subject is [15]. In particular, we make use of the concept of perfect Bayesian equilibrium (PBE). A PBE, like a subgame perfect Nash equilibrium, is a strategy profile – or specification of each player's strategy – such that no player can increase her expected payoff by unilaterally deviating from her PBE strategy at any point in the game. The important feature that distinguishes a PBE from a subgame perfect equilibrium is that, in this context, the access point maintains and refines conditional probability distributions on the random type-variables that describe the client.

In general, a strategy specification of a dynamic Bayesian game is a mapping from a player's type, and a player's information set to an action, or in a mixed strategy, to a distribution among possible actions [15]. Assuming players have "perfect recall," the information set is the history of actions the player has observed. In our model, if the game reaches slot t, then the history is completely specified by the previous prices charged. Thus, a pure strategy for an access point is simply a price sequence. An access point's mixed strategy, or behavior strategy, is a probability distribution on prices to charge in each slot t dependent upon the prices actually charged in earlier slots 1,..., $t-1$. For the client, a strategy is specified as a mapping from its type and information set to an acceptance decision, or to a probability of accepting.

III. WEB BROWSING MODEL

In this section we model a client browsing the web. The client's utility is proportional to the length of time T

that she gets to browse the web, but her utility saturates after the maximum intended session length τ is reached:

$$F(T, \tau) = U \cdot \min(T, \tau). \tag{1}$$

The client's type is specified by her utility per slot U and intended session length τ. The client knows the values for U and τ while the access point just knows their distributions.

Theorem 1: Consider a web browser client with utility defined by expression (1). Suppose that U and τ are independent and finite-mean, and U has a continuous distribution. Then the following strategy profile is a PBE equilibrium.

- The client connects or remains connected in slot t iff $t \leq \tau$ and $p_t \leq U$. (We refer to this as the "myopic strategy.")
- The access point charges a non-decreasing sequence of prices $\{p_t\}$ such that

$$p_t \in \arg\max_p pP(U \geq p).$$

Note that Theorem 1 suggests that an access point should pick its prices by looking at just the prior distribution of U. In fact, an important corollary to Theorem 1 is that it is a PBE for the access point to pick a single maximizing value of $pP(U \geq p)$, say p^* and charge the fixed price $p_t = p^*$ in all time slots.

This is somewhat surprising, because whenever a myopic client accepts price p_t, the access point can refine its conditional distribution of U by lower bounding it by p_t. One might have expected that an access point might want to try charging a higher price than p_t after learning that the client's utility is at least p_t. Theorem 1 states this intuition is not correct.

We now prove Theorem 1.

Proof: First, we find the access point's optimal counter strategy to a client playing the "myopic strategy." A pure strategy for the access point can be specified by a sequence of nonnegative prices $\{p_t\}_{t=1}^{\infty} \in \Re_+^{\{1,2,...\}}$ to charge at each time slot $t \in \{1, 2, ...\}$. The access point wishes to choose his sequence of prices to maximize his expected revenue, as shown in the following expression:

$$J_1^a(\{p_t\}) = \sum_{t=1}^{\infty} p_t P(U > \max_{u \in \{1,...,t\}} p_u) P(\tau \geq t). \tag{2}$$

In the notation $J_1^a(\{p_t\})$, the a superscript signifies that it is the expected revenue (objective) of the access point, the 1 subscript indicates that it is the objective from slot 1 onward, and the $(\{p_t\})$ signifies that the objective is a function of the access point's price sequence. Expression (2) reflects that a client will be connected in slot t if and only if her utility per slot U is greater than the

current price p_t as well as all previous prices, $p_1, ..p_{t-1}$, and if the client's intended session length τ is not less than t. We see that we can find a maximizing sequence of expression (2) even if we restrict ourselves to non-decreasing sequences. This is because for any sequence $\{\tilde{p}_t\}$ for which there exists a u such that $\tilde{p}_u < \tilde{p}_{u-1}$, we can define a new non-decreasing sequence $\{p_t\}$ with $p_t = \max(\tilde{p}_t, ..., \tilde{p}_1)$ and we would have $J_1^a(\{p_t\}) \geq J_1^a(\{\tilde{p}_t\})$. The same idea expressed in terms of the situation – an access point that has seen a myopic client accept a price p_t knows that he can charge at least p_t in slot $t + 1$ without risking that he charges more than the client is willing to pay. For convenience, we define S^+ to be the set of non-decreasing price sequences, that is $\{p_t\} \in S^+$ if $\{p_t\} \in \Re_+^{\infty}$ and $p_{t+1} \geq p_t \ \forall t \in \{1, 2, ...\}$. The access point wishes to choose a positive sequence of prices to maximize his expected revenue, as shown in the following expression:

$$\max_{\{p_t\} \in S^+} J_1^a(\{p_t\})$$

$$= \max_{\{p_t\} \in S^+} \left[\sum_{t=1}^{\infty} p_t P(U > p_t) P(\tau \geq t) \right]. \tag{3}$$

Because U and τ are finite mean, one can substitute their Markov bounds into expression (3) to show that the access point's expected payoff against a myopic client is bounded [17]. Each term in the summation of expression (3) is a function of a different price, p_t, so the entire sum can be maximized by independently maximizing each term in the summation. We note that $\arg\max_p pP(U \geq p)$ is non-empty because $y(p) = pP(U \geq p)$ is a continuous, nonnegative function, with $y(0) = 0$, and $\lim_{p \to \infty} y(p) = 0$, and thus must achieve a maximum on $[0, \infty)$. So if the access point chooses each p_t such that $p_t \in \arg\max_p pP(U \geq p)$, with $\{p_t\} \in S^+$, then the access point maximizes his expected payoff (3).

Now looking at the client's side, it is easy to see that the myopic strategy is a best response to an access point that never lowers prices.

Because these strategies are best responses to each other, they constitute a (Bayesian) Nash equilibrium strategy profile. Now we will verify that the strategy profile is a PBE – that the strategy profiles remain best responses to each other in any continuation game, beginning at an arbitrary slot s. A client facing nondecreasing prices in this game will of course face nondecreasing prices in any continuation game starting at slot s, thus a client that expects nondecreasing prices should stick to the myopic strategy in the continuation game beginning at slot s. An access point that expects his client to be myopic should choose his prices in the continuation

game to maximize

$$J_s^a(\{p_t\}_{t=s}^\infty)$$
$$= \sum_{t=s}^\infty \left[p_t\, P\left(U > \max_{u \in \{s,...,t\}} p_u \,\middle|\, U > p_{s-1} \right) \right.$$
$$\left. \times P(\tau \ge t | \tau \ge s) \right]. \qquad (4)$$

For any price sequence $\{\tilde{p}_t\}_{t=s}^\infty$ which has prices that are less than p_{s-1}, we see that $J_s^a(\{\max(\tilde{p}_t, p_{s-1})\}_{t=s}^\infty) \ge J_s^a(\{\tilde{p}_t\}_{t=s}^\infty)$. Thus the access point can maximize its reward to go by selecting its continuation game prices to be no smaller than p_{s-1}. Thus assuming $p_u \ge p_{s-1}$ for all $u \ge s$ we may write,

$$J_s^a(\{p_t\}_{t=s}^\infty)$$
$$= \frac{1}{P(U > p_{s-1})} \sum_{t=s}^\infty \left[p_t\, P\left(U > \max_{u \in \{s,...,t\}} p_u \right) \right.$$
$$\left. \times \frac{1}{P(\tau \ge s)} P(\tau \ge t) \right]. \qquad (5)$$

Note that expression (5) has a structure that parallels expression (2), with the exception of the scaling factors $1/P(U > p_{s-1})$ and $1/P(\tau \ge s)$ which have no dependence on the prices chosen from slot s forward. Thus the same argument that was used to show that expression (2) is maximized with a nondecreasing sequence with elements in $\arg\max_p pP(U \ge p)$ can be used to show that expression (5) is also maximized with a nondecreasing sequence with elements in $\arg\max_p pP(U \ge p)$. Therefore we have shown that the access point strategy described in the statement of Theorem 1 is remains a best response to a myopic client in any continuation game. ∎

Our web browser result is similar to results shown in other contexts in the economics literature. For example, in [16] the authors show that under certain assumptions that it is not more profitable for a seller to condition pricing on the past behavior of the customer.

A. Web Browsing Examples

We now consider a few specific examples of the web browsing model. In the first example, suppose that the client's per slot valuation, U, is distributed uniformly on $[0, 1]$. Then the unique maximizer of $pP(U > p)$ would be 0.5, and thus the PBE specified by Theorem 1 is that the access point charges 0.5 in all slots, and that the client plays a myopic strategy. In the second example, suppose that the game started with U distributed uniformly on $[0.5, 1]$. The unique maximizer of $pP(U > p)$ would be 0.5, and the access point should charge 0.5 in all time slots in PBE. We see that in this second example, the distribution of U, is the same as the conditional distribution of U in the first example, after a client accepts the price of 0.5 in slot 1. Thus the second example is equivalent to a continuation game of the first example, so it should be expected that the equilibrium prices should be the same. Finally, if U is distributed uniformly on $[a, b]$, we find that the access point should charge $\max(b/2, a)$ in each slot, according to the PBE described by Theorem 1.

B. Uniqueness of the PBE

Theorem 1 shows that a strategy profile with the client playing a myopic strategy is a PBE, however it does not show that this strategy profile is the unique PBE. In the special case that the intended session length τ is bounded, we can show that the myopic strategy is the unique client strategy in PBE.

Lemma 1: Suppose τ is distributed on $\{1, ..., n\}$, and U is finite mean, then the following characterizes all PBE:
- The client follows a myopic strategy.
- The access point charges a non-decreasing sequence of prices $\{p_t\}$ such that

$$p_t \in \arg\max_p pP(U \ge p).$$

Proof: In slot n, the client's dominant strategy is the myopic strategy. This is simply because there is no future after slot n, so the client should myopically maximize her expected return in slot n

After deleting the client's dominated strategies, the access point's dominant counter strategy in slot n is to charge at least $\max(p^*, p_1, ... p_{n-1})$. Where $p^* = \min(\arg\max_p pP(U \ge p))$

In slot $n-1$, the client can anticipate the access point's action in slot n. The client knows that $p_n \ge p_{n-1}$. Facing non-decreasing prices on $\{n-1, n\}$, the client's dominant strategy in slot $n-1$ is the myopic strategy.

Suppose we have shown that for each slot $u \ge t$, the access point charges at least $\max(p^*, p_1, ... p_{u-1})$, and the client plays the myopic strategy in slot u. In slot $t-1$, the client knows that prices $\{p_{t-1}, ..., p_n\}$ will be a non decreasing sequence. The client's dominant strategy in slot $t-1$ is therefore the myopic strategy. The access point thus charges at least $\max(p^*, p_1, ... p_{t-2})$.

By induction, the client plays the myopic strategy in all time slots. The access point's best response is to charge a non-decreasing sequence of prices $\{p_t\}$ such that $p_t \in \arg\max_p pP(U \ge p)$. ∎

C. Multiple Hops

Having considered a single-hop model in which an access point sells access directly to a client, we now
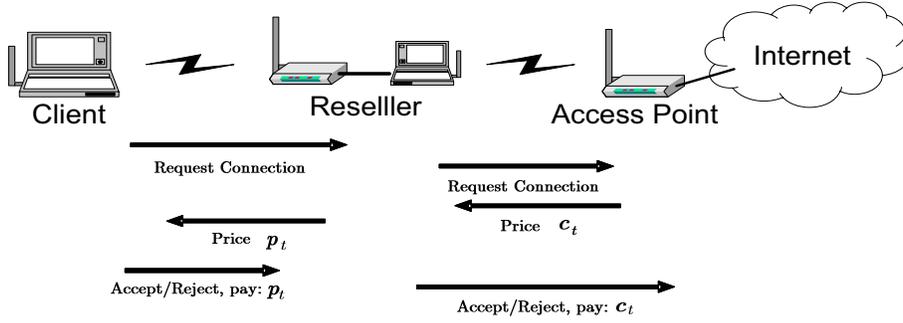
Fig. 2.   Multi-hop scenario: Client, Reseller, and Root Access Point.

consider a scenario where a "root" access point sells service to a reseller, which in turn sells the service to an end client with a web browser utility. This situation arises when a client is not within communicating range of an access point with a wired Internet connection, and instead needs an intermediate node, or reseller in our terminology, to act as a relay.

The situation is depicted in Figure 2. As shown in Figure 2, the client tries to begin a session by sending a request for service to the reseller. In order to serve the client's request, the intermediate node sends its own request for service to the "root" access point that has a wired Internet connection. The root access point passes the reseller a price for the first slot, c_1, which the reseller can either accept or reject, and as in the single-hop game, the game ends upon the first rejection. Before deciding to accept or reject the c_1 price, the reseller sends its first slot-price, called p_1, to the client. If the client accepts p_1, the reseller would accept the c_1 price from the root access point. If the client rejects p_1, and assuming that the reseller had no other use for connectivity with the access point than to serve the client, the reseller would reject the c_1 price and the game would end. If the client and reseller accept their offers in slot t, the game continues into slot $t+1$ with the root access point choosing a price c_{t+1} to offer the reseller, and the reseller choosing a price p_{t+1} to offer the client.

As in the single-hop model, in this game there is also a notion of intended session length, which is the length of time after which the client stops earning utility from remaining connected. The intended session length is a random variable τ with its sample value known to the client, and only its distribution known to the other parties. Thus the client's utility is described by expression (1), where T is the number of slots the client chooses to remain connected. The clients payoff is simply her utility minus what she pays, $F(T, \tau) - \sum_{t=1}^{T} p_t$. The re-seller's payoff is simply the difference of the payments he receives and the payments he makes, $\sum_{t=1}^{T} (p_t - c_t)$, while the access

point's payoff is the sum of all payments he receives from the reseller, $\sum_{t=1}^{T} c_t$.

As in the single-hop game, a client's pure strategy is specified as a mapping from her type and information set to an acceptance decision. A pure strategy for the access point is specified as a sequence of prices $c_1, c_2, ...$ to charge in each time slot.

In general, a pure strategy for the reseller is a mapping from prices he has been charged $c_1, ..., c_t$, as well as the prices the reseller charged in the past, $p_1, ..., p_{t-1}$, to a price to charge in the current slot, p_t. For convenience, we denote the reseller's history as

$$\mathbf{h}_t^r \stackrel{\triangle}{=} \left(\{c_u\}_{u=1}^{t}, \{p_u\}_{u=1}^{t-1} \right).$$

Thus a pure strategy for the reseller is a specification of the functions $p_t(\mathbf{h}_t^r)$.

Having identified what the strategy spaces for the three players look like in general, we now identify a specific strategy profile that is a PBE.

Theorem 2: The following strategy profile is a PBE:

- The client follows a myopic strategy, connecting iff $t \leq \tau$ and $p_t \leq U$.
- The reseller picks a function $p^*(c)$ that satisfies the properties:

$$p^*(c) \in \arg\max_p (p - c) P(U > p) \quad (6)$$
$$p^*(c') \geq p^*(c) \ \forall c' > c \quad (7)$$

and charges the price $p_t(\mathbf{h}_t^r) = p^*(c_t)$ in slot t.
- The access point charges a non decreasing price sequence $\{c_t\}$ with

$$c_t \in \arg\max_c \left[c \cdot P(U > p^*(c)) \right]$$

Note that in the equilibrium reseller strategy profile detailed in Theorem 2, the choice of price is just a function of his current cost c_t. This is significant because in general the reseller's price strategy can in general depend on the past costs $c_1, ..., c_{t-1}$ as well. We now

state and prove a Lemma that we will later use in the proof of Theorem 2.

Lemma 2: There exists a function $p^*(c)$ that satisfies properties (6) and (7).

Proof: Define

$$y_c(p) \triangleq (p - c)P(U > p).$$

$y_c(p)$ is a continuous function with $y_c(c) = 0$, $y_c(p) \geq 0$ for $p > c$, and has $\lim_{p \to \infty} y_c(p) = 0$. Thus, $y_c(p)$ must achieve a maximum value somewhere on $[c, \infty)$. Thus $\arg\max_p y_c(p)$ is a non empty set.

We find $p^*(c)$ by construction. Set

$$p^*(c) = \min\left(\arg\max_p y_c(p)\right).$$

Note that that $p^*(c)$ is well defined for all $c > 0$ because $\arg\max_p y_c(p)$ is non empty. It remains for us to show that p^* is monotonic non-decreasing, which we will do by contradiction. Suppose it were not monotone non-decreasing. Then their exists $(c^l, c^h) : c^l < c^h$ with $p^*(c^h) < p^*(c^l)$. For convenience, we define $p^l = p^*(c^h)$ and $p^h = p^*(c^l)$ so that $p^l < p^h$. It must be that

$$(p^h - c^l)P(U > p^h) > (p^l - c^l)P(U > p^l) \quad (8)$$

or else p^h would not be the lowest valued maximizer of $y_{c^l}(\ )$. It also must be that

$$(p^l - c^h)P(U > p^l) \geq (p^h - c^h)P(U > p^h) \quad (9)$$

or else p^l would not be the lowest valued maximizer of $y_{c^h}(\ )$. Combining (8) and (9) we have

$$\frac{p^l - c^h}{p^h - c^h} > \frac{p^l - c^l}{p^h - c^l}. \quad (10)$$

But expression (10) implies

$$p^l c^l + c^h p^h < c^h p^l + p^h c^l.$$

Regrouping terms we get

$$p^h(c^h - c^l) < p^l(c^h - c^l),$$

which is a contradiction.

Thus the $p^*(c)$ we constructed must be monotone non-decreasing, and thus we have found a function that satisfies (6) and (7). ∎

With Lemma 2 proved, we may now prove Theorem 2.

Proof: We begin the proof of Theorem 2 by showing that if an access point charges the reseller a sequence of prices $\{c_t\} \in S^+$ where S^+ is the set of non-decreasing sequences, and if the client plays the myopic strategy, then it is a best response for the reseller to use the strategy $p_t = p^*(c_t)$ where the function $p^*(c)$ has the properties (6) and (7).

The reseller should choose a mapping from history $\mathbf{h}_t^r$ to price p_t to maximize expected reward:

$$J_1^r\left(\{p_t(\mathbf{h}_t^r)\}; \{c_t\}\right)$$
$$= \sum_{t=1}^{\infty}\left[(p_t - c_t)P\left(U > \max_{u \in \{1, \dots, t\}} p_u\right)\right.$$
$$\left. \times P(\tau \geq t)\right]. \quad (11)$$

The $; \{c_t\}$ notation in $J_1^r\left(\{p_t(\mathbf{h}_t^r)\}; \{c_t\}\right)$ signifies that the reseller objective function is dependent upon the resller's assumption of the access point's strategy, which for pure strategies can be specified as the sequence $\{c_t\}$. Define a modified objective

$$\tilde{J}_1^r(\{p_t\}; \{c_t\}) \triangleq \sum_{t=1}^{\infty}(p_t - c_t)P(U > p_t)P(\tau \geq t). \quad (12)$$

Note that

$$J_1^r|_{\{p_t\} \in S^+}(\{p_t\}; \{c_t\}) = \tilde{J}_1^r|_{\{p_t\} \in S^+}(\{p_t\}; \{c_t\}) \quad (13)$$

and

$$J_1^r(\{p_t\}; \{c_t\}) \leq \tilde{J}_1^r(\{p_t\}; \{c_t\}). \quad (14)$$

Now suppose the reseller picks a function $p^*(c)$ satisying properties (6) and (7) and uses the strategy $p_t(\mathbf{h}_t^r) = p^*(c_t)$. Then for every $\{c_t\} \in S^+$, $p_t(\mathbf{h}_t^r) = p^*(c_t)$ maximizes expression (12) because the sum is separable and can be maximized term by term. Furthermore, expressions (13) and (14) imply that $p_t(\mathbf{h}_t^r) = p^*(c_t)$ also maximizes expression (11) for all possible $\{c_t\} \in S^+$. Thus the strategy $p_t(\mathbf{h}_t^r) = p^*(c_t)$ is a best response to a myopic client and access point that does not decrease prices. Next we show that this strategy remains the best response in all continuation games. Suppose the reseller reaches slot s having used the strategy $p_t(\mathbf{h}_t^r) = p^*(c_t)$, the reseller wishes to maximize the expected reward to go:

$$J_s^r\left(\{p_t(\mathbf{h}_t^r)\}; \{c_t\}\right)$$
$$= \sum_{t=s}^{\infty}\left[(p_t - c_t)P\left(U > \max_{u \in \{s, \dots, t\}} p_u \,\middle|\, U > p_{s-1}\right)\right.$$
$$\left. \times P(\tau \geq t | \tau \geq s)\right]. \quad (15)$$

From expression 15, we see that an access point can maximize expected reward to go by considering only prices greater than or equal to p_{s-1}, thus we may write:

$$J_s^r\left(\{p_t(\mathbf{h}_t^r)\}; \{c_t\}\right)$$
$$= \frac{1}{P(U > p_{s-1})}\sum_{t=s}^{\infty}\left[(p_t - c_t)P\left(U > \max_{u \in \{s, \dots, t\}} p_u\right)\right.$$
$$\left. \times \frac{1}{P(\tau \geq s)}P(\tau \geq t)\right]. \quad (16)$$

The objective in expression (16) differs from expression (11) only by factors $1/P(U > p_{s-1})$ and $1/P(\tau \geq s)$ which have no dependence on the prices chosen from slot s forward. Thus, the same arguments used to show that the strategy $p_t(\mathbf{h}_t^r) = p^*(c_t)$ is a best response starting from time slot 1 can be used to show that it is also a best response starting from time slot s.

Next we look at the access point strategy. We assume the client is myopic, and that the reseller uses the strategy $p_t(\mathbf{h}_t^r) = p^*(c_t)$. The access point wishes to maximize his expected revenue

$$
\begin{aligned}
& J_1^a(\{c_t\}; \{p_t(\mathbf{h}_t^r)\}) \\
& = \sum_{t=1}^{\infty} c_t P(U > \max_{u \in \{1,...,t\}} p^*(c_u)) P(\tau \geq t).
\end{aligned}
$$

Because $p^*(c)$ is monotone, we can invert it and define the random variable $V = p^{*-1}(U)$. Now, the access point's objective becomes

$$
\begin{aligned}
& J_1^a(\{c_t\}; \{p_t(\mathbf{h}_t^r)\}) \\
& = \sum_{t=1}^{\infty} c_t P(V > \max_{u \in \{1,...,t\}} c_u) P(\tau \geq t). \quad (17)
\end{aligned}
$$

Expression (17) is identical in structure to expression (2), which describes the expected payoff of an access point in the single hop model. Thus from the access point's perspective, the multi-hop scenario in which the access point sells to a reseller which in turn sells to a client with a utility per slot U is equivalent to selling directly to a client with its utility per slot distributed like V. Thus the same argument used in the proof of Theorem 1 to verify the best response of the access point in the single-hop scenario can be re-used here to show that a best response of the access point would be to pick a sequence of prices $\{c_t\}$ with $c_t \in \arg\max_c P(V > c)$.

Finally we observe that if the root access point charges a nondecreasing price, and that the reseller's price is a static function of access point price, then the client's best response is to play a myopic strategy. This is because the client's best response to non-decreasing prices is a myopic strategy. ∎

1) Example of Multi-hop Equilibrium: The sharing of revenue between the reseller and access point is similar to that of the leader and follower in the classic Stackelberg competition game. In the Stackelberg game, a leader and follower firms successively decide how much of a good to produce, and then the market price of the good is determined by the sum of the two firms productions and a demand function [18]. In the model developed here the access point, analogous to the "leader," chooses a price c_t, while the "follower," or reseller, chooses a markup over the access point's price, $p_t - c_t$. The client's

probability of accepting is a function of the sum of the leader's and follower's choices.

In the Stackleberg game, the leader makes at least as much revenue as the follower in equilibrium. By analogy, we can observe that the access point's share of the revenue should be greater than or equal to the reseller's revenue in equilibrium. For instance, supposing that the client's distribution of U were uniform on $[0, 1]$, in the PBE characterized by Theorem 2, the access point would charge 0.50 in each time slot, while the client would charge 0.75. In this example the access point would have have an expected revenue twice that of the access point.

IV. FILE TRANSFER MODEL

Here we model a situation in which a client is downloading a file, and the client must remain connected for the entire duration of the file, or in our terminology the intended session length, to earn any utility for the file. The client's utility function has the form:

$$
F(t, \tau) = \begin{cases} 0 & \text{if } t < \tau \\ U\tau & \text{if } t = \tau \end{cases} \quad (18)
$$

The client's type is determined by the two random variables τ and U, where τ is the intended session length, in this case the length of the file, and U is the client's utility per unit of file length.

Theorem 3: Suppose the client has a file transfer utility function as in expression (18), with U a continuous random variable on $[l, h]$ with $0 \leq l < h$, and the session length τ, distributed on $\{1, ..., n\}$. Both U and τ have sample values *known* to the client, and *unknown* to the access point. We also assume that U and τ are finite mean, and that U is continuously distributed. Then the following characterizes all perfect Bayesian equilibria:

- The client plays a "pessimistic" strategy: The client accepts in slot $t < \tau$ iff $p_t = 0$. When $t = \tau$, the client connects if she had been connected in all of the previous slots, and $p_t < U\tau$. (She never connects if $p_t > U\tau$ but may connect if $p_t = U\tau$.)
- The access point charges:

$$
p_t = \begin{cases} 0 & \text{if } t < t^* \\ u^* t^* & \text{otherwise} \end{cases} \quad (19)
$$

where $(u^*, t^*) \in \arg\max_{(u,t)} utP(U > u, \tau = t)$

Proof: The proof uses backwards induction, and iterated deletion of dominated strategies. We begin by showing that clients with intended session length n follow the pessimistic strategy.

Suppose the client has an intended session length of n and utility parameter $U = u$. We will refer to such a client as a type (u, n) client. When the game reaches

slot n, a type n client's dominant strategy is to accept any price less than nu, because doing so would earn her a payoff greater than if she refused to finish her transfer in the last slot. Because U is lower-bounded by l, the access point should charge at least nl in slot n. More specifically, after deleting his client's dominated strategies, and deleting the access point's own dominated strategies, the access point's remaining strategies involve charging at least nl.

Knowing that a access point will charge at least nl in the last slot, clients of types $([l, l + \frac{\epsilon}{n}), n)$ will be unwilling to pay more than ϵ in any slot before their last slot, n. Otherwise, such clients would predictably end up paying at least $ln + \epsilon$ and would finish the game with negative payoff. We refer to this as an ϵ-pessimistic strategy.

Now suppose a client of unknown type were to stay connected up until slot n, and in at least one slot prior to n did pay ϵ or more. The access point would deduce that the client can not be of types $([l, l + \frac{\epsilon}{n}), n)$, because such clients play an ϵ-pessimistic strategy. Therefore, the access point deduces that the client's type must be in the range$([l + \frac{\epsilon}{n}, h], n)$ and charges at least $ln + \epsilon$, knowing that all clients in the type range would be compelled to pay.

Knowing that a access point would charge at least $nl + \epsilon$ in the last slot, clients of type $([l, l + \frac{2\epsilon}{n}, h), n)$ also play an ϵ-pessimistic strategy.

Supposing that we have shown by iterated deletion of dominated strategies that an access point would charge at least $nl + t\epsilon$ in the last slot. Consequently, clients of types $([l, l + \frac{(t+1)\epsilon}{n}), n)$ play an ϵ-pessimistic strategy. A access point in slot n facing a client of unknown type can eliminate the possibility that his client's type is in the range $([l, l + \frac{(t+1)\epsilon}{n}), n)$, and thus charge at least $nl + (t+1)\epsilon$.

By induction clients of types $([l, h], n)$, play an ϵ-pessimistic strategy, and this is true for any $\epsilon > 0$. The only strategy for which this is true for all $\epsilon > 0$ is the "pure" pessimistic strategy of accepting a maximum price of 0 in slots prior to the final slot of the file transfer.

We now prove the induction step. Suppose we have shown that Clients of Types $([l, h], j)$ through types $([l, h], n)$ follow the pessimistic strategy. We show it for Type $([l, h], j - 1)$:

Suppose that a type $([l, h], j - 1)$ client does not play the pessimistic strategy by accepting a nonzero price in a slot with index less than $j - 1$. When the game reaches slot $j - 1$, the access point can deduce that the client is of type $j - 1$ or greater, and that clients of type j

or greater would have already quit the game. Thus, the access point knows he faces a type $j - 1$ client. Knowing his client's file ends in slot $j - 1$, the access point can charge $(j - 1)l$ and be assured that the client will be compelled to pay. Thus clients of types $([l, l + \frac{\epsilon}{n}), j - 1)$ will be unwilling to pay more than ϵ in any slot before their last slot, $j - 1$. Continuing this argument inductively in exactly the same way as we did to show that clients of type $([l, h], n)$, are ϵ pessimistic we can show that clients of type $([l, h], j - 1)$ are ϵ pessimistic. The only strategy that is ϵ pessimistic for all $\epsilon > 0$ is the 'pure" pessimistic strategy of accepting a maximum price of 0 in slots prior to the final slot of the file transfer. Thus, clients of type $([l, h], j - 1)$ are pessimistic.

By induction, clients of all types play the pessimistic strategy.

Access Point counter strategy:

An access point facing pessimistic clients has only one chance to charge nonzero prices. The access point charges according to expression (19) where (u^*, t^*) in $\arg\max_{(u,t)} utP(U > u, \tau = t)$. Note that one can show that $\arg\max_{(u,t)} utP(U > u, \tau = t)$ is nonempty by using the fact that U and τ are finite mean. ∎

A. Inefficiency of the File Transfer Model Equilibrium

The equilibrium described by Theorem 3 is very inefficient. If the client's intended session length is greater than t^*, in equilibrium the access point would be forced to charge 0 in the slots prior to t^*, and then at time t^*, the client will refuse to pay the access point's price. If the client's intended session length is less than t^*, in equilibrium the client will finish her file download without paying anything. Only when the client's intended session length is exactly t^* will the access point earn any revenue.

V. BAYESIAN MODEL

In Section III, we found that when a client has a web browsing utility as described by expression (1) that an access point charges a constant price in PBE. In Section IV, we discovered that when a client has a file transferor utility as described by expression (18), that an access point does not charge a constant price in PBE. In this section, we study the case in which the access point does not know if his client is a file transferor or a web browser. We model this situation by assuming that the access point begins the game knowing the prior probability, x, that the client is a file transferor. We call this combined model simply the Bayesian Model.
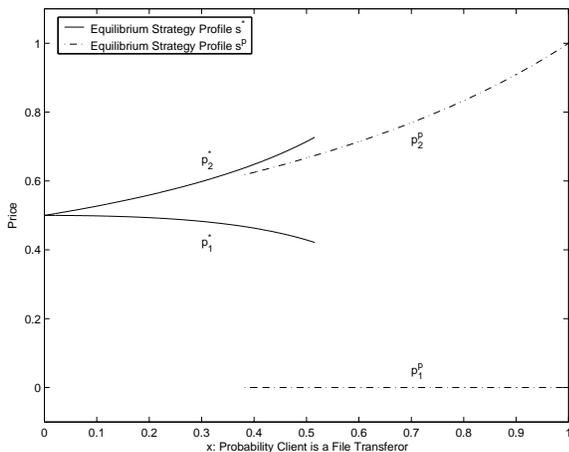
Fig. 3. PBE prices for the 2 time slot Bayesian game described in the statement of Lemma 3.

Although, we would like to find a general solution to the Bayesian Model without making any assumptions about the probability distributions of τ, the intended session length, and U, the utility per slot, we are most interested in knowing whether the price is constant in PBE. When x is 0, the Bayesian Model is equivalent to a "pure" web browsing model, so constant price is a PBE by Theorem 1. When x is 1, the Bayesian Model is equivalent to a "pure" file transfer model, where we know that constant price is not a PBE. Based on these facts, one might hypothesize that when x has a small enough value, constant price is a PBE. We will show by studying an example, in which we choose specific distributions for τ and U, that this hypothesis is not correct. We state the example, and the results of its analysis in Lemma 3 which follows.

Lemma 3: Suppose that the client is a file transferor (FT) with probability x and a web browser (WB) with probability $1-x$. The access point knows the value of x, while the client knows her true type. Also suppose that clients of both types have an intended session length of $\tau = 2$, and this is known to both parties. The utility per slot U is uniformly distributed on $[0, 1]$. The client knows the sample value of U while the access point (AP) knows only the distribution. The utility functions for WB and FT type clients are given by expressions (1) and (18) respectively. Then the following three assertions are true:

1) The strategy profile (pair of player strategies), $s^* = (s_{AP}^*, s_C^*)$ is a PBE $\forall x \in [0, 0.516]$, where the access point strategy, s_{AP}^*, and the type dependent client strategy, s_C^*, are defined as follows:

- s_{AP}^*: Charge the price sequence $\{p_1^*, p_2^*\}$ in the 2 slots of the game, where the prices p_1^*

and p_2^* are dependent on x as :

$$p_1^* = \frac{4-5x}{2(1-x)(4-x)}, \quad p_2^* = \frac{4-3x}{2(1-x)(4-x)}.$$

- $s_C^*(WB)$: (WB clients) Connect in slot 1 iff $p_1 \leq U$. Connect in slot 2 iff connected in slot 1 and $p_2 \leq U$. Note that this is a myopic strategy.
- $s_C^*(FT)$: (FT clients) Connect in slot 1 iff $p_1 + \hat{p}_2 \leq 2U$ where $\hat{p}_2$, intuitively the price the client expects in slot 2, is equal to p_2^*. Connect in slot 2 iff connected in slot 1 and $p_2 \leq 2U$.

2) The strategy profile, $s^p = (s_{AP}^p, s_C^p)$ is a PBE $\forall x \in [\frac{3-\sqrt{5}}{2} \approx 0.382, 1]$, where the player strategies are defined as:

- s_{AP}^p: Charge the price sequence $\{0, \frac{1}{2-x}\}$.
- $s_C^p(WB)$: The myopic strategy,

$$s_C^p(WB) = s_C^*(WB).$$

- $s_C^p(FT)$: The pessimistic strategy – connect in slot 1 iff $p_1 = 0$ and connect in slot 2 iff connected in slot 1 and $2U \geq p_2$.

(Note that we chose the superscript p in s^p to signify that FT clients are pessimistic in this strategy profile.)

3) For $x > 0$ there are no PBE in which the AP charges a constant price ($p_1 = p_2$).

The prices of the two equilibrium strategy profiles described by Lemma 3 are shown in Figure 3.

Proof: We begin by showing that s^* is a PBE. First we consider whether the client's strategy s_C^* is a best response to the access point strategy s_{AP}^*. The access point prices are nondecreasing in s_{AP}^*, and we have seen that a web browser's best response to nondecreasing prices is a myopic strategy, so $s_C^*(WB)$ is a best response. Similarly, when the AP plays s_{AP}^*, playing $s_C^*(FT)$ gives the highest possible payoff to a FT client for all possible values of U. Furthermore, a FT client does not benefit by unilaterally deviating in the continuation game beginning in slot 2.

Next we consider whether s_{AP}^* is a best response to s_C^*. We begin by writing an expression for the access point revenue $R(p_1, p_2)$ assuming that clients play s_C^*:

$$R(p_1, p_2) = p_1 \left[(1-x)G(p_1) + xG\left(\frac{p_1+p_2^*}{2}\right) \right] +$$
$$p_2 \left[(1-x)G\left(\max[p_2, p_1]\right) + xG\left(\max\left[\frac{p_1+p_2^*}{2}, \frac{p_2}{2}\right]\right) \right]$$

where $G(p) = P(U > p) = \max((1-p), 0)$. $R(p_1, p_2)$ is piecewise quadratic in (p_1, p_2). By breaking up $R(p_1, p_2)$ into quadratic functions on different regions of $\Re^2$, finding the maxima on each region, and then finding the global maximum across all of the regional maxima, it can

be shown that (p_1^*, p_2^*) is the unique maximizing value of $R(p_1, p_2)$ for $x \in [0, 0.516]$, where 0.516 is a decimal approximation to the root of a 6th order polynomial. Thus (p_1^*, p_2^*) is the AP's best response to clients that play s_C^*, for $x \in [0, 0.516]$. (Note that for values of x larger than ≈ 0.516, s_{AP}^* is not a best response because the AP can earn greater expected revenue by charging a different set of prices than (p_1^*, p_2^*).) To see that the access point has no incentive to deviate from s_{AP}^* in the continuation game beginning at slot 2, we can use Bayes' Rule to find that the AP's expected revenue in slot 2, given the client accepted price p_1^* in slot 1. After a few transformations we have,

$$J_2^a(p_2) = \frac{R(p_1^*, p_2)}{p_1^* \left[(1-x)G(p_1^*) + xG\left(\frac{p_1^* + p_2^*}{2}\right) \right]} - p_1^*$$

which is maximized when $p_2 = p_2^*$.

Next we show that s^p is a PBE. Under the prices of s_{AP}^p, clients following s_C^p connect whenever connecting will result in a positive payoff, and do not connect otherwise. Thus s_C^p is a best response to s_{AP}^p, and furthermore the a client's best response in the continuation game beginning in slot 2 is to not deviate from s_C^p.

With clients playing s_C^p, an AP can charge 0 in the first slot, and keep the FT clients connected, or choose a nonzero price and earn revenue from only the WB clients. If the AP chooses the latter option, then he should maximize expected revenue from WB clients, which he can do by charging $\frac{1}{2}$ in both slots, earning him an expected revenue of $(1 - x)\frac{1}{2}$. If the AP chooses 0 for its slot 1 price, then both FT and WB clients would be potential customers in the 2nd slot, and the AP's optimal 2nd slot price is found by maximizing

$$p_2(1-x)P(U \geq p_2) + p_2 x P\left(U \geq \frac{p_2}{2}\right)$$

which happens at $p_2 = \frac{1}{2-x}$, earning an expected revenue of $\frac{1}{4-2x}$. The latter option of charging $\{0, \frac{1}{2-x}\}$, which is the same as the s_{AP}^p strategy defined in the statement of Lemma 3, earns more expected revenue than the option of charging $\frac{1}{2}$ in each slot for $x \in [\frac{3-\sqrt{5}}{2}, 1]$. Thus s_{AP}^p is a best response to to s_C^p for $x \in [\frac{3-\sqrt{5}}{2}, 1]$.

Though we have shown that the strategy profiles s^* and s^p are PBE for particular ranges of x, we have not yet shown that there is not another PBE where the access point charges a fixed price $p_1 = p_2$. Let us suppose the AP does follow the strategy of charging $p_1 = p_2 = p$, and look for a contradiction. The best response for both FT and WB clients is to follow a strategy of accepting in slot 1 whenever $U > p$. When a client accepts in first slot, in the next slot the AP faces a client with $U > p$, and a posterior probability, x of being a FT type. In the continuation game beginning at slot 2, a FT client has a dominant strategy of accepting whenever $2U > p_2$, while a WB client has a dominant strategy of accepting whenever $U > p_2$. Thus an access point that wishes to charge constant price should choose that price to maximize

$$2p(1-x)P(U \geq p) + pxP(U \geq p)[1 + P\left(U \geq \tfrac{p}{2} | U \geq p\right)]$$

which occurs at $p' = \frac{1}{2}$. However in the continuation game beginning at slot 2, the access point maximizes revenue by maximizing

$$p_2(1 - x)P(U \geq p_2 | U \geq p) + p_2 x P\left(U \geq \tfrac{p_2}{2} | U \geq p\right)$$

which occurs at $p_2' = \max(p, \frac{1}{2-x})$. For $x > 0$, $p_2' > p'$, thus in slot 2 the access point wants to deviate from the constant price strategy and charge the higher price p_2'. Thus we have shown that constant price is not a PBE for $x > 0$. ∎

We make a few observations about whether the equilibria identified in Lemma 3 are unique. When $x = 0$, the model is equivalent to a pure web browsing model, and Lemma 1 applies, telling us that the PBE is unique, and price is constant. The equilibrium strategy profile s^* covers this case because $p_1^* = p_2^* = \frac{1}{2}$ when $x = 0$. When $x = 1$, the model is equivalent to a pure file transferor model, and we can use Theorem 3 to say that the unique PBE is one in which FT clients are pessimistic. This PBE is the same as s^p from Lemma 3. For $0 < x < 1$ it is possible, and indeed likely, that there are other PBE that we have not identified here. However, as we stated and showed in Lemma 3, when $x \in (0, 1]$ there is no PBE for which the access point charges constant price.

Though price is not constant in PBE for $x > 0$, the PBE described by s^* has the access point charging prices that are "reasonable" in that they there is a nonzero probability that a client's single slot utility exceeds each price. This is in contrast to the PBE of the pure file transfer model in which the access point, in which the access point tries to charge for the whole value of a file in a single slot. In the section that follows, we study whether prices are "reasonable" when file lengths have unbounded distributions.

A. Unbounded Length

In Section IV we found that when file transferor clients have file lengths picked from a bounded distribution, clients are pessimistic and that the access point does not charge a constant price. In Section V, we studied a Bayesian Model that combines the file transfer and web browsing models, and in an example where file length is bounded, we found that the access point price

is not constant. In this section we study what happens in PBE when the file length is not picked from a bounded distribution. In Theorem 4 which follows, we find that in a Bayesian Model in which intended session length has an unbounded distribution, it is not a PBE for the access point to charge "reasonable" prices in every slot. Where we say a price is "reasonable" if it has a nonzero probability of being less than a client's single slot utility. Note that Theorem 4 implies that it is not a PBE for an access point to charge constant price. In the statement of Theorem 4 we make modest assumptions about the distribution of τ which hold, for example, for the geometric distribution.

Theorem 4: Suppose that:

- The intended session length τ is distributed on $\{1, 2, ...\}$.
- With probability $x > 0$ the client has a file transferor (FT) utility function: $F(t, \tau) = U\tau \cdot 1(t \geq \tau)$.
- With probability $1-x$, the client has a web browser (WB) utility function: $F(t, \tau) = U \min(t, \tau)$.
- U is positive, continuously distributed, and independent of τ, and whether the client is of type FT or WB.
- The distribution of τ is independent of whether the client is of type FT or WB, and there exists constants $\alpha > 0$ and $\delta > 0$ such that for all $t > 0$,

$$P(\tau = t | \tau \geq t) > \delta$$
$$E[\tau - t | \tau \geq t] < \alpha.$$

Then in PBE the access point (AP) price in each time slot is not bounded by any constant h for which $P(U > h) > 0$.

Proof: Suppose the strategy profile $s = (s_{AP}, s_C)$ is a PBE in which the access point prices are never more than h. For convenience, we define $\epsilon = P(U > h)$. If the game reaches slot t, the AP can consider deviating from s_{AP} at slot t by charging a price of ht to exploit FT clients whose transfers are finishing in slot t. The one-step expected revenue for such a deviation is

$$htP(\text{Client} = FT, \tau = t, U > h | \tau \geq t, d_{t-1} = \text{C}) \quad (20)$$

where the notation $\{d_{t-1} = \text{C}\}$ signifies the event in which the client is connected after slot $t - 1$. Using Bayes' Rule, the independence of the type variables, and the fact that FT clients will always be connected in slot $t - 1$ when $U > h$ and $\tau \geq t$, we write the following

inequalities to bound expression (20):

$$P(\text{Client} = FT, \tau = t, U > h | \tau \geq t, d_{t-1} = \text{C})$$
$$= \frac{P(\text{Client} = FT, \tau = t, U > h, d_{t-1} = \text{C} | \tau \geq t)}{P(d_{t-1} = \text{C} | \tau \geq t)}$$
$$\geq P(U > h, \text{Client} = FT, \tau = t | \tau \geq t)$$
$$= xP(U > h)P(\tau = t | \tau \geq t) = x\epsilon\delta. \quad (21)$$

Thus, using expression (21) we see that the deviation of charging ht at time t earns at least $htx\epsilon\delta$.

Now we attempt to characterize the expected revenue an AP earns for sticking to strategy s_{AP} at time t. Under s_{AP}, the expected revenue to go from time t forward, which we call $J_t^a(s_{AP})$, cannot be more than if AP receives h in all the remaining time slots of the client's session length. Thus

$$J_t^a(s_{AP}) < hE[(\tau - t + 1) | \tau \geq t, d_{t-1} = \text{C}]. \quad (22)$$

To derive a bound on expression (22), we again use the fact that an FT client following s_C always connects in slot t if $U > h$ and $\tau \geq t$. We find that $\forall j \geq t$:

$$P(\tau = j | \tau \geq t, d_{t-1} = \text{C})$$
$$= \frac{P(\tau = j, d_{t-1} = \text{C} | \tau \geq t)}{P(d_{t-1} = \text{C} | \tau \geq t)}$$
$$\leq \frac{P(\tau = j | \tau \geq t)}{P(U > h)} = \frac{1}{\epsilon} P(\tau = j | \tau \geq t). \quad (23)$$

Now we use expression (23) to bound expression (22):

$$hE[(t - \tau + 1) | \tau \geq t, d_{t-1} = \text{C}]$$
$$= h \sum_{j=t}^{\infty} (j - \tau + 1) P(\tau = j | \tau \geq t, d_{t-1} = \text{C})$$
$$\leq \frac{h}{\epsilon} \sum_{j=t}^{\infty} (j - \tau + 1) P(\tau = j | \tau \geq t)$$
$$\leq \frac{h}{\epsilon} E[(t - \tau + 1) | \tau \geq t] < \frac{h}{\epsilon}(1 + \alpha).$$

Thus, $J_t^a(s_{AP}) \leq \frac{h}{\epsilon}(1 + \alpha)$. For $t > \frac{1}{\epsilon^2 \delta x}(1 + \alpha)$, the one step reward for deviating and charging ht at time t exceeds the upper-bound on the expected reward for maintaining s_{AP}. Thus s_{AP} cannot be a best response to s_C, and thus s is not a PBE. ∎

VI. RESULT SUMMARY AND IMPLICATIONS TO P2P MODEL OF WIFI PRICING

We have seen that if the client is a web browser, with a utility function that grows linearly with connection duration, it is a PBE for the access point to charge a constant price, p^* in each time slot. Though the value of p^* depends on the distribution of U, the utility per unit time, the fact that constant price is a PBE is true for

any distributions of type variables U and τ (the intended session length) so long as U and τ are finite mean and independent. The result even extends to a multi-hop case where an access point sells to a reseller which in turn sells to a client, as we saw in Section III-C. These results suggest that if a client has a web browsing utility function, that we could expect an access point to charge constant price without third party supervision, and without the need for contracts. An architecture based on micropayments, like the architecture we explored in Section I-D would likely lead to a functioning market.

However, we found in Section IV that if the client has a file transferor utility, where utility has a step with respect to time, then the access point price is not constant in PBE. Further more, when the file length has a bounded distribution, clients are pessimistic, and the PBE can be very inefficient in terms of social welfare. The access point prices are not constant even when there is only a probability of the client being a file transferor, as we saw in the Bayesian Model of Section V. When the Bayesian Model is modified for clients that have an unbounded intended session length, it remains true that prices are not constant, and furthermore it is not a PBE for the access point to charge "reasonable" prices in every slot. Where a "reasonable" price is one in which there is a nonzero probability of the client's one slot utility exceeding it.

Despite the disappointing properties of the equilibria in the file transferor cases, we feel that the P2P charging model is viable, if the granularity of the slots is chosen judiciously. For one reason, we feel that the web browsing model is a more realistic representation of a typical client's utility. Most mobile users are probably interested in browsing the web, using e-mail, or perhaps downloading small files. As long as the email spool or small files can be downloaded in less than one slot, then the step discontinuities in client utility disappear when looked at using the discrete time scale of the game. However, the slot size should not be made too large, because the client might not feel comfortable paying in advance for a large block of time. From a game theory perspective, an access point with marginal cost per slot δc would be tempted to take a slot payment and then not serve the client if $p_t > (p_t - \delta c)E[(\tau - t)|\tau > t]$. Here $E[(\tau - t)|\tau > t]$ is the expected remaining intended session length, which for finite mean τ approaches 1 as the slot size increases (Assuming τ is a discretized version of an underlying continuous random variable). We feel that a slot size of about 1 minute should achieve both objectives.

With any choice of time slot length, users will on occasion download files that take longer than one time slot to complete. To address this issue we look to file transfer software that already exists today that allow clients to resume an interrupted file transfer at some later time [20]. A client using such software does get partial utility for partial files, and thus her utility function would look more like that of our web-browser model than of our file transferor model.

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